Information Systems
(Informationssysteme)

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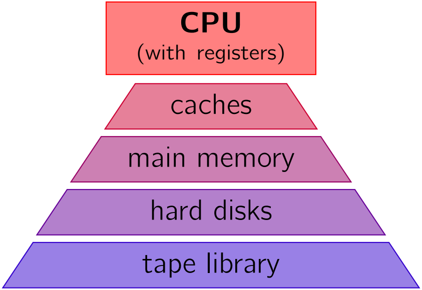
Part IX

B-Trees

Memory Hierarchy



capacity	latency
bytes	< 1 ns
kilo-/megabytes	< 10 ns
gigabytes	70–100 ns
terabytes	3–10 ms
petabytes	varies

- fast, but expensive and small, memory close to CPU
- larger, slower memory at the periphery
- Try to **hide latency** by using the fast memory as a **cache**.

Latency vs. Bandwidth

"Slow" memory typically means **high latency**.

Example: Samsung HD642JJ Hard Drive (640 GB, SATA 3)

- rotational speed: 7200 rpm
- sequential read bandwidth: $\approx$ 106 MB/s ($\nearrow$ `hdparm -t`)
- random access time: 15.2 ms (measured)

✎ **Time it takes to read 1,024 random 4 kB blocks?**

Ways to Improve I/O Performance

The latency penalty is hard to avoid.

However:

- Throughput can be increased rather easily by exploiting **parallelism**.
- **Idea:** Use multiple disks and access them in parallel.

> **TPC-C: An industry benchmark for OLTP**
>
> The current number one system (Oracle 11g RAC on SPARC) uses
>
> - 11,040 flash drives (24 GB each) and 720 hard drives **(!)** (plus drives for OS, etc.),
> - connected with 8 Gbit Fibre Channel,
> - yielding 30 tpmC ($\approx$ 60 M transactions per minute).

Consequences of the Bandwidth $\leftrightarrow$ Latency Gap

To combat the latency problem:

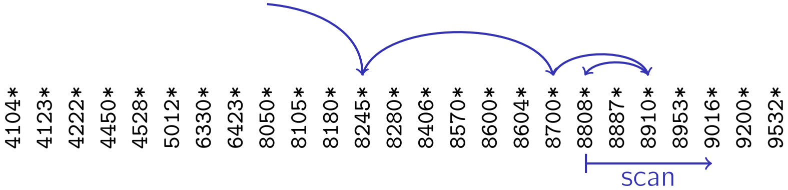
1. Databases access and organize the disk with a **page granularity**.
   - Read larger chunks to amortize high latency.
   - Page size: at least 4 kB, better more; up to $\approx$ 64 kB.
2. Use **sequential access** and/or aggressive **prefetching** (read-ahead).
   - But must read **many** pages ahead to actually avoid penalty.

Finding a Needle in a Haystack

```
SELECT  *
  FROM  CUSTOMERS
 WHERE  ZIPCODE BETWEEN 8800 AND 8999
```

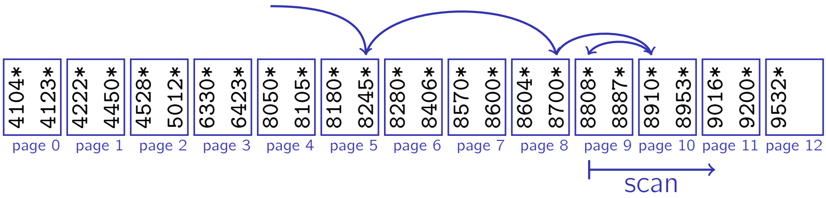
To answer this query, we could

1. **sort** the table on disk (in `ZIPCODE` order).
2. To answer queries, then use **binary search** to find first qualifying tuple, and **scan** as long as `ZIPCODE` $<$ 8999.



$k*$ denotes the full data record with search key k.

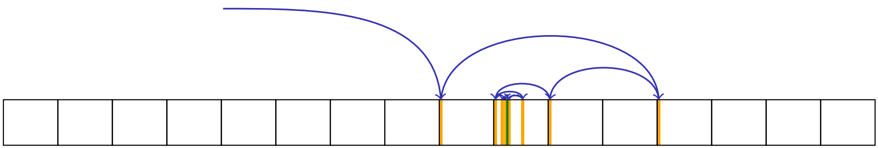
Ordered Files and Binary Search



✔ Need to read only $\log_2(\#\,\mathit{tuples})$ to find the first match.

✗ Need to read about as many **pages** for this.
(The whole point of binary search is that we make far, unpredictable jumps. This largely defeats prefetching.)

Binary Search and Database Pages

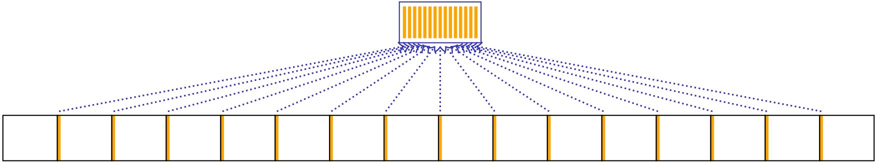


Observations:

- Make rather **far jumps initially**.
  - → For each step read **full page**, but inspect only **one record**.
- Last $\mathcal{O}(\log_2 \mathit{pagesize})$ steps stay **within one page**.
  - → I/O cost is used much more efficiently here.

Binary Search and Database Pages

Idea: "Cache" those records that might be needed for the first phase.

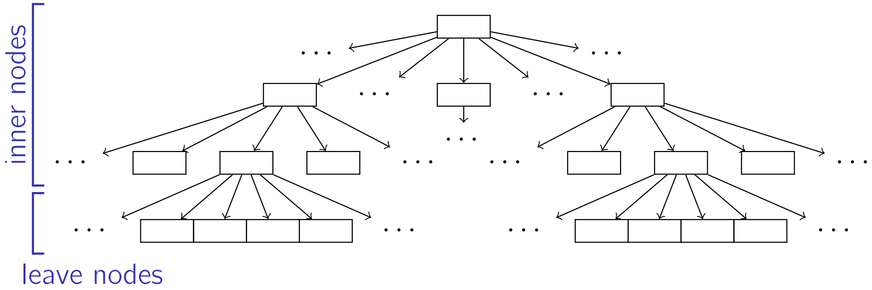


→ If we can keep the cache **in memory**, we can find **any** record with just a **single I/O**.

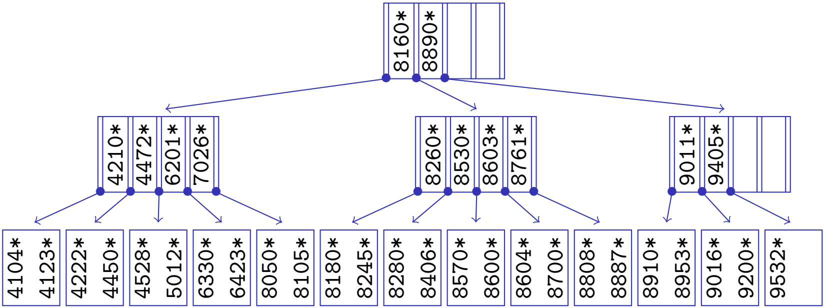
✎ **Is this assumption reasonable?**

What if my data set is really large?

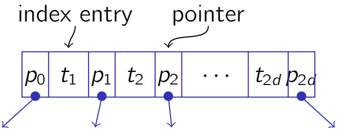
- "Cache" will span many pages, too.
  (In practice, we'll organize the cache just like any other database object.)
- Thus: **"cache the cache"** → hierarchical "cache"



- **B-trees** are essentially such a "hierarchical cache."



- All nodes are the size of a page
  - → hundreds of entries per page
  - → large fanout, low depth
- Search effort: $\log_{\textit{fanout}}(\#\ \textit{tuples})$



↗ Rudolf Bayer and Edward McCreight. *Organization and Maintenance of Large Ordered Indexes*, Acta Informatica 1(3), 1972.

B-Trees

Each B-tree node contains

- A set of **index entries**, which include
  - the value of a **search key** (*e.g.*, `4711`) and
  - **"associated information"** (indicated by $*$) (either a full data tuple or a reference to a tuple).
- A set of **child pointers**, pointing to a child page of the B-tree.

Each tree node (except the root) contains **at least** d and **at most** $2d$ index entries ("minimum 50 % full"; on previous slide: $d = 2$).

$\rightarrow$ We call d the **order** of the B-tree.

$\rightarrow$ In practice, d is **large** (few hundreds).

B-trees are **balanced** at all times.

Searching a B-Tree

Function: `tree_search` (k, *node*)
if matching $*_i$ found on *node* **then**
    return $*_i$;
if *node* is a leaf node **then**
    return not found;
switch k **do**
    case $k < k_0$ **do**
        return `tree_search` (k, p_0);
    case $k_i < k < k_{i+1}$ **do**
        return `tree_search` (k, p_i);
    case $k_{2d} < k$ **do**
        return `tree_search` (k, p_{2d});

- Invoke with *node* = root node.
- Note that B-trees are an **ordered** index structure.
  → Support equality and range predicates

B-Tree Modifications

Goal: Keep B-tree **balanced** at all times.[14]

✎ **Why is this desirable?**

Thus: Define routines for **insertion** and **deletion** that leave the B-tree properties intact.

[14] *I.e.*, every root-to-leaf path must have the same length.

Inserting into a B-Tree

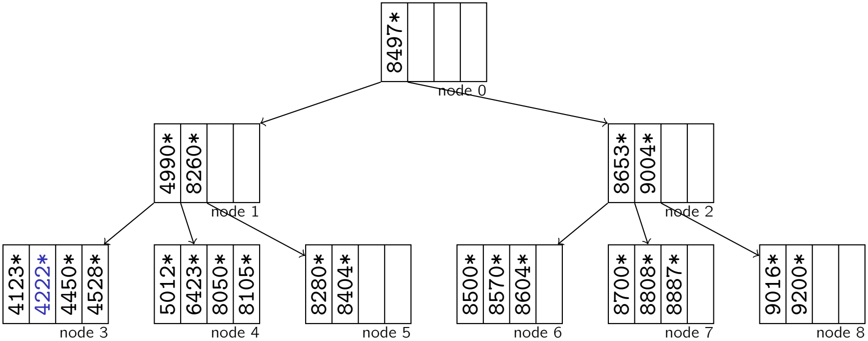
Sketch of the **insertion procedure** for entry $k*$:

1. **Find leaf page** n where we would expect the entry for k.
2. If n has **enough space** to hold the new entry (*i.e.*, at most $2d - 1$ entries in n), **simply insert** $k*$ into n.
3. Otherwise node n must be **split** into n and n' and a new **separator** has to be inserted into the parent of n.

   Splitting happens recursively and may eventually lead to a split of the root node (increasing the tree height).

   → B-trees grow at the root, not at the leaves!

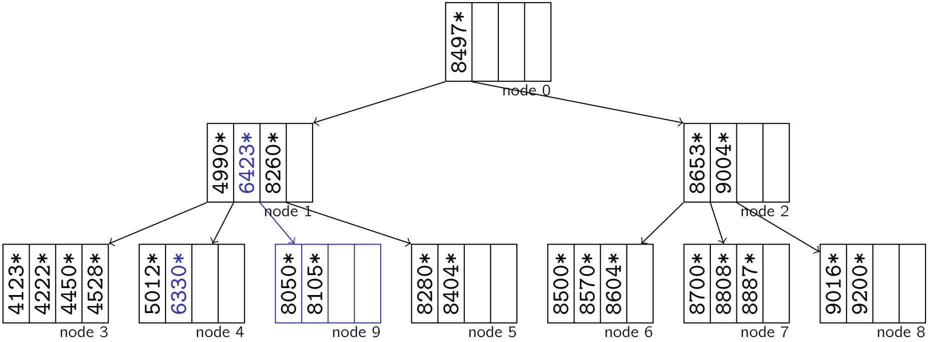
B-Tree Insert: Examples (Insert without Split)



Insert new entry with key 4222.

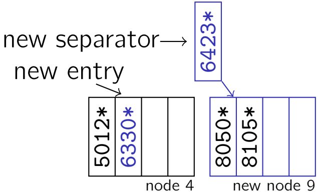
→ Enough space in node 3, simply insert.

B-Tree Insert: Examples (Insert with Leaf Split)

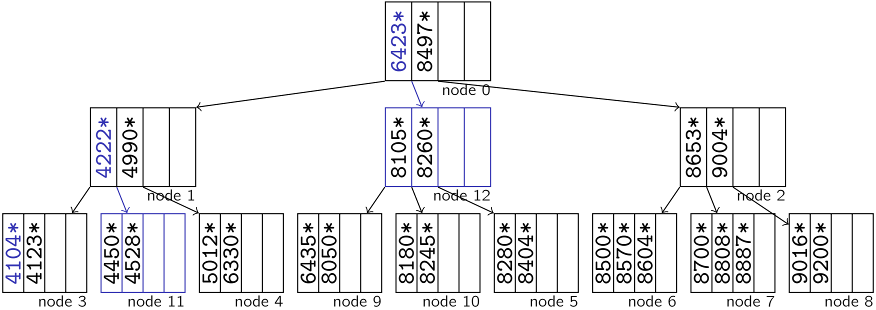


Insert key 6330.

- → Must **split** node 4.
- → **Middle entry** goes into node 1.

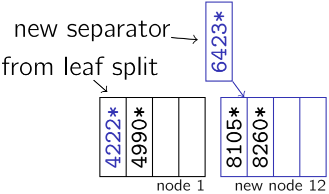


B-Tree Insert: Examples (Insert with Inner Node Split)



After 8180, 8245, 6435 insert key 4104.

- → Must **split** node 3.
- → Node 1 overflows → split it
- → **New separator** goes into root



Insert: Root Node Split

- Splitting starts at the leaf level and continues upward as long as index nodes are fully occupied.
- Eventually, this can lead to a split of the **root node**:
  - Split like any other inner node.
  - Use the separator to create a **new root**.
- The root node is the **only** node that may have an occupancy of less than 50 %.
- This is the **only** situation where the tree height increases.

✎ **How often do you expect a root split to happen?**

Keys and Tuples

A B-tree maintains **key values** together with **"associated information"**.

The "associated information" $*$ can be

Full Data Tuples

The B-tree becomes the mechanism to organize the table data

- → The table is **physically ordered** according to the key attribute.
- → We call this a **clustered index** or an **index-organized table**.
- → There can be at most one clustered index per table.

Pointers to Actual Tuples

These pointers are also called **record identifiers** or **RIDs**.

- → Most systems use $\langle$*pageno*, *pos. within page*$\rangle$ to encode RIDs.
- → Such indexes are also called **secondary indexes**.
- → There can be arbitrarily many secondary indexes.

Many systems (*e.g.*, DB2) only support the latter index type.

B-Trees $\rightarrow$ B$^+$-trees

Key to the efficiency of B-trees is their **high fanout**.

high fanout $\rightarrow$ low tree depth $\rightarrow$ fast root-to-leaf navigation

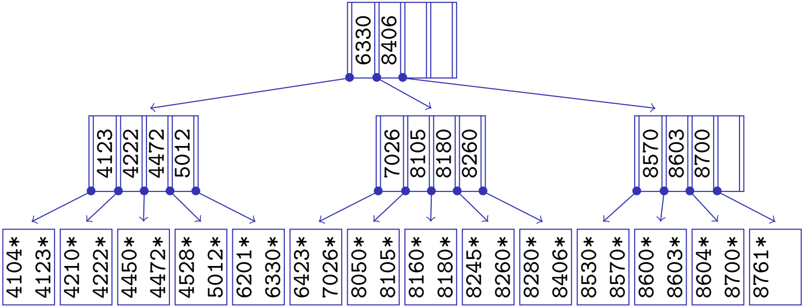
This gives incentive to **maximize fanout**:

- $\rightarrow$ Do **not** store $*$ in **inner nodes**
  (Rather use that space to increase d / store more keys.)
- $\rightarrow$ Inner nodes are then used for root-to-leaf navigation **only**.
- $\rightarrow$ For every data tuple, there is on leaf-level index entry.
- $\rightarrow$ The resulting index structure is then called **B$^+$-tree**.

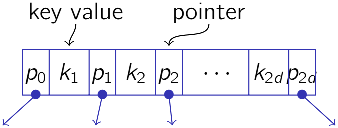
Real systems today always use B$^+$-trees.

- $\rightarrow$ When database people say "B-tree," they typically mean "B$^+$-tree."

B^+-trees



- Inner nodes do **not** store tuples or RIDs
  - → only used to navigate to leaves
  - → higher fanout, lower depth
- Only leaves contain (references to) tuple data (indicated here with ∗)



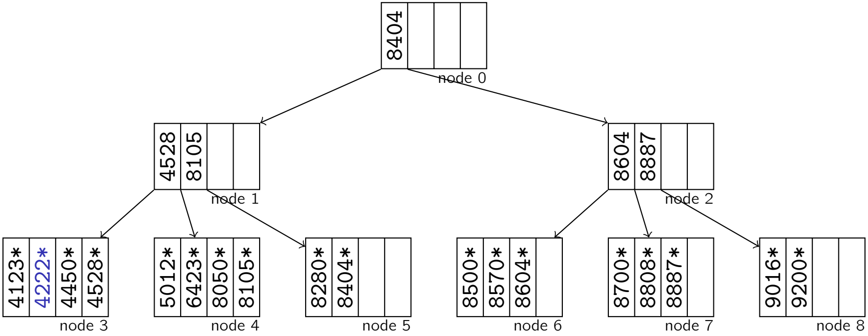
Searching a B^+-tree

```
Function: search (k)
return tree_search (k, root);
```

```
Function: tree_search (k, node)
if node is a leaf then
    return node;
switch k do
    case k ≤ k_0 do
        return tree_search (k, p_0);
    case k_i < k ≤ k_{i+1} do
        return tree_search (k, p_i);
    case k_{2d} < k do
        return
          tree_search (k, p_{2d});
```

- All searches now navigate to a leaf node.
  - → Makes search effort also more predictable.
- Function `search` (k) returns a pointer to the leaf node that contains potential hits for search key k.

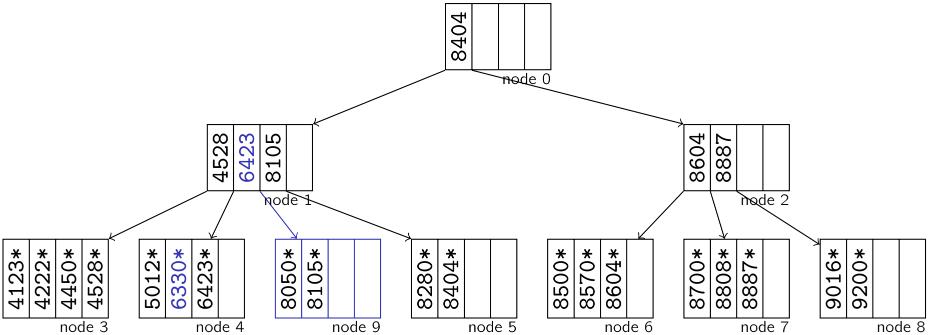
B^+-tree Insert: Examples (Insert without Split)



Insert new entry with key 4222.

→ Enough space in node 3, simply insert.
(Same as in B-tree)

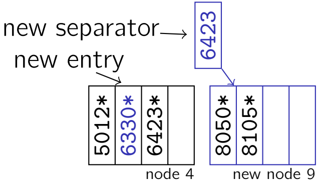
B^{+}-tree Insert: Examples (Insert with Leaf Split)



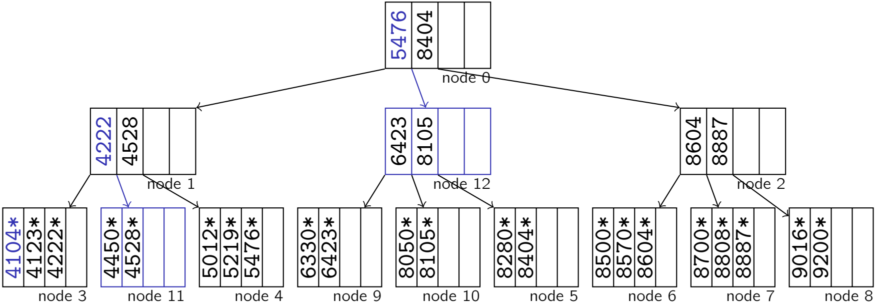
Insert key 6330.

→ Must **split** node 4.

→ **New separator** goes into node 1.
But **keep** entry in node 4**!**



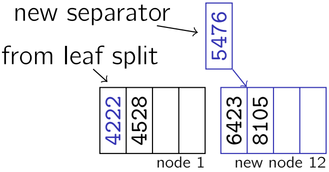
B$^+$-tree Insert: Examples (Insert with Inner Node Split)



After 5219, 5476, insert key 4104.

- → Must **split** leaf node 3.
- → Inner node 1 overflows → split it
- → **New separator** goes into root

Splitting the **inner node** works analogously to B-tree.



B^+-tree Insertion Algorithm

Function: `tree_insert` (k, rid, $node$)

if $node$ is a leaf **then**
- **return** `leaf_insert` (k, rid, $node$);

else
- **switch** k **do**
  - **case** $k \leq k_0$ **do**
    - $\langle sep, ptr \rangle \leftarrow$ `tree_insert` (k, rid, p_0);
  - **case** $k_i < k \leq k_{i+1}$ **do**
    - $\langle sep, ptr \rangle \leftarrow$ `tree_insert` (k, rid, p_i);
  - **case** $k_{2d} < k$ **do**
    - $\langle sep, ptr \rangle \leftarrow$ `tree_insert` (k, rid, p_{2d});

  see `tree_search ()`
- **if** sep is **null then**
  - **return** $\langle$**null**, **null**$\rangle$;
- **else**
  - **return** `split` (sep, ptr, $node$);

Function: `leaf_insert` (k, *rid*, *node*)

if another entry fits into *node* **then**

  insert $\langle k, rid \rangle$ into *node* ;

  return $\langle \mathbf{null}, \mathbf{null} \rangle$;

else

  allocate new leaf page p ;

  take $\left\{\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle\right\}$:= entries from *node* $\cup \{\langle k, ptr \rangle\}$

    leave entries $\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_{d+1}^+, p_{d+1}^+ \rangle$ in *node* ;

    move entries $\langle k_{d+2}^+, p_{d+2}^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle$ to p ;

  return $\langle k_{d+1}^+, p \rangle$;

Function: `split` (k, *ptr*, *node*)

if another entry fits into *node* **then**

  insert $\langle k, ptr \rangle$ into *node* ;

  return $\langle \mathbf{null}, \mathbf{null} \rangle$;

else

  allocate new leaf page p ;

  take $\left\{\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle\right\}$:= entries from *node* $\cup \{\langle k, ptr \rangle\}$

    leave entries $\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_d^+, p_d^+ \rangle$ in *node* ;

    move entries $\langle k_{d+2}^+, p_{d+2}^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle$ to p ;

    set $p_0 \leftarrow p_{d+1}^+$ in *node*;

  return $\langle k_{d+1}^+, p \rangle$;

B^+-tree Insertion Algorithm

```
Function: insert (k, rid)
⟨key, ptr⟩ ← tree_insert (k, rid, root);
if key is not null then
    allocate new root page r;
    populate n with
        p_0 ← root;
        k_1 ← key;
        p_1 ← ptr;
    root ← r ;
```

- `insert` (k, *rid*) is called from outside.
- Note how leaf node entries point to RIDs, while inner nodes contain pointers to other B^+-tree nodes.

Example

Example: Webserver access log (`people.inf.ethz.ch`)

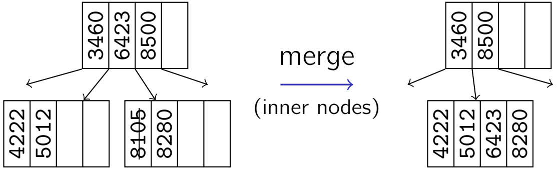
- table cardinality: 11 million tuples (710K data pages)
- distinct IP addresses: 181,628 (stored as `CHAR(15)`)
- database: IBM DB2 9.7

B$^+$-tree on IP addresses:

- 25,151 index pages total:
  - 1 root node
  - 110 second-level nodes; average fanout: 230
  - 25,040 leaf-level nodes: 1–77 keys per node

Deletion

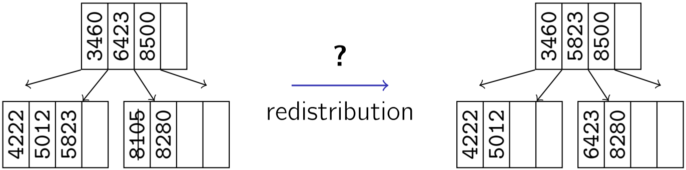
- If a node is sufficiently full (*i.e.*, contains at least $d+1$ entries, we may simply remove the entry from the node.
  - Note: Afterward, **inner nodes** may contain keys that no longer exist in the database. This is perfectly legal.
- **Merge** nodes in case of an **underflow** ("undo a split"):



  - "Pull" separator into merged node.

Deletion

It's not quite that easy. . .



- Merging only works if **two** neighboring nodes were 50 % full.
- Otherwise, we have to **re-distribute**:
  - "rotate" entry through parent
- Redistribution is **complex** and **expensive**.

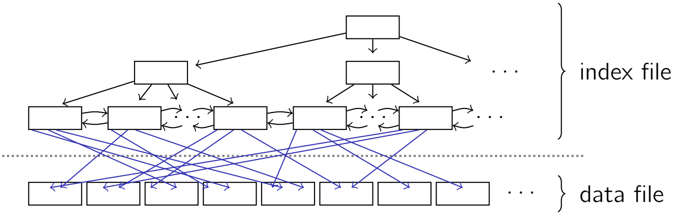
  → Real systems usually do not implement deletion "by the book."

B^{+}-trees in Real Systems

- Actual systems often avoid the cost of merging and/or redistribution, but relax the minimum occupancy rule.
- *E.g.*, **IBM DB2 UDB**:
  - The `MINPCTUSED` parameter controls when the system should try a leaf node merge ("on-line index reorg").
  - Inner nodes are never merged
    ($\rightarrow$ need to do full table reorg for that).
- To improve **concurrency**, systems sometimes only **mark** index entries as deleted and physically remove them later (*e.g.*, IBM DB2 UDB "type-2 indexes")

  $\rightarrow$ Resulting index entries are also called **ghost records**.

A typical situation (for a secondary B^{+}-tree) looks like this:



✎ **What are the implications when we want to execute**

`SELECT * FROM CUSTOMERS ORDER BY ZIPCODE` **?**

Composite Keys

B^+-trees can (in theory[15]) be used to index everything with a defined **total order**, *e.g.*:

- integers, strings, dates, . . . , and
- **concatenations** thereof (based on **lexicographical order**).

E.g., in most SQL dialects:

```
CREATE INDEX ON TABLE CUSTOMERS (LASTNAME, FIRSTNAME);
```

A useful application are, *e.g.*, **partitioned B-trees**:

- Leading index attributes effectively **partition** the resulting B^+-tree.

↗ G. Graefe: Sorting And Indexing With Partitioned B-Trees. *CIDR 2003*.

[15]Some implementations won't allow you to index, *e.g.*, large character fields.

Partitioned B-trees

```
CREATE INDEX ON TABLE STUDENTS (SEMESTER, ZIPCODE);
```

✎ **What types of queries could this index support?**