Architecture and Implementation of Database Systems (Summer 2020)

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Part III

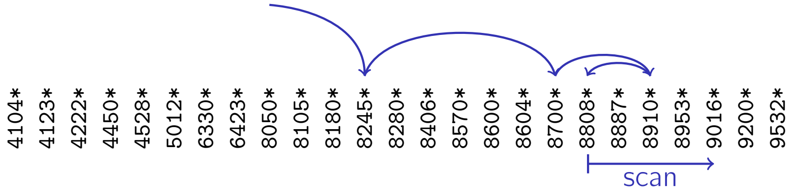
Indexing

```
SELECT  *
  FROM  CUSTOMERS
 WHERE  ZIPCODE BETWEEN 8800 AND 8999
```

How could we prepare for such queries and evaluate them efficiently?

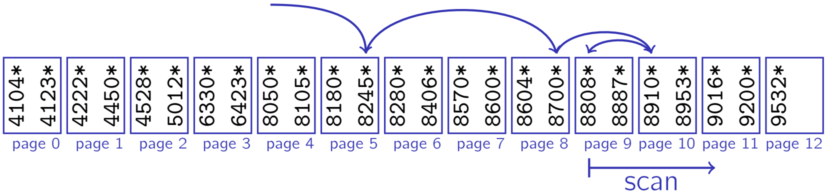
We could

1. **sort** the table on disk (in `ZIPCODE` order).
2. To answer queries, then use **binary search** to find first qualifying tuple, and **scan** as long as `ZIPCODE` $<$ 8999.



$k*$ denotes the full data record with search key k.

Ordered Files and Binary Search

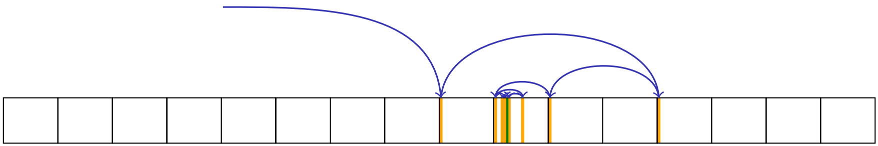


✔ We get **sequential access** during the **scan phase**.

We need to read $\log_2(\#\ tuples)$ tuples during the **search phase**.

✗ We need to read about as many **pages** for this.
(The whole point of binary search is that we make far, unpredictable jumps. This largely defeats prefetching.)

Binary Search and Database Pages

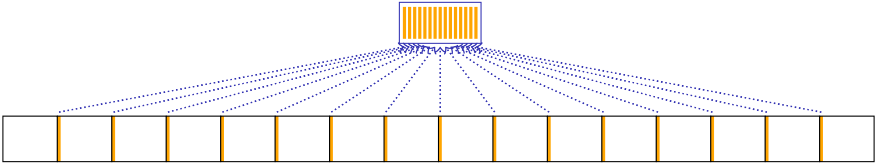


Observations:

- Make rather **far jumps initially**.
  - $\rightarrow$ For each step read **full page**, but inspect only **one record**.
- After $\mathcal{O}(\log_2 \textit{pagesize})$, search stays **within one page**.
  - $\rightarrow$ I/O cost is used much more efficiently here.

Binary Search and Database Pages

Idea: "Cache" those records that might be needed for the first phase.

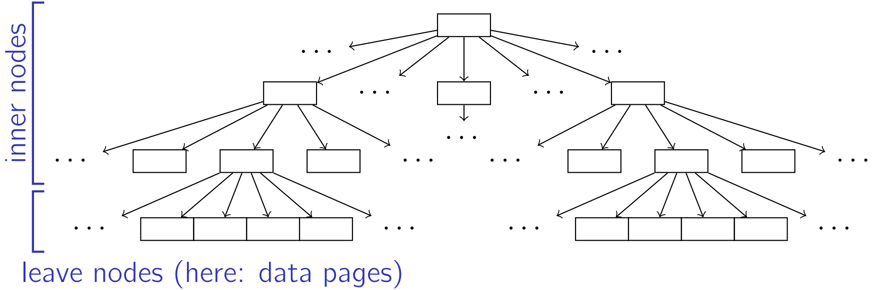


→ If we can keep the cache **in memory**, we can find **any** record with just a **single I/O**.

✎ **Is this assumption reasonable?**

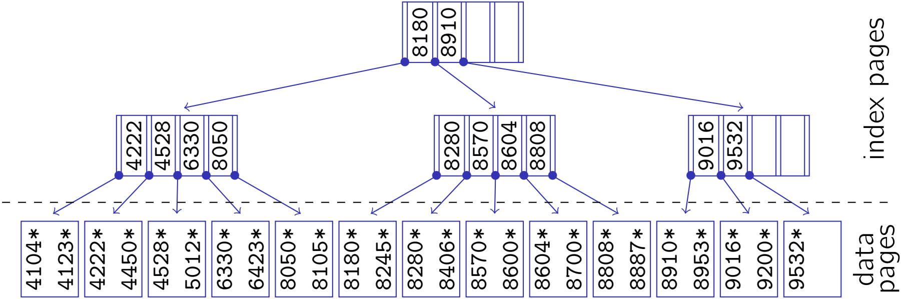
What if my data set is really large?

- "Cache" will span many pages, too.
  (In practice, we'll organize the cache just like any other database object.)
- Thus: **"cache the cache"** → hierarchical "cache"

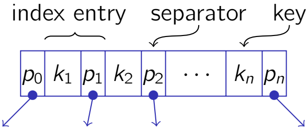


ISAM—Indexed Sequential Access Method

Idea: Accelerate the search phase using an index.



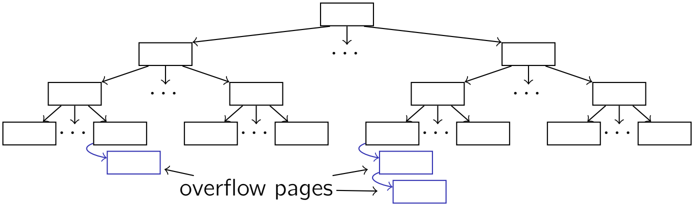
- All nodes are the size of a page
  - → hundreds of entries per page
  - → large fanout, low depth
- Search effort: $\log_{fanout}(\#\ tuples)$



ISAM Index: Updates

ISAM indexes are inherently **static**.

- **Deletion** is not a problem: delete record from data page.
- **Inserting** data can cause more effort:
  - If space is left on respective leaf page, insert record there (*e.g.*, after a preceding deletion).
  - Otherwise, **overflow pages** need to be added.
    (Note that these will **violate** the sequential order.)
  - ISAM indexes **degrade** after some time.



Remarks

- Leaving some free space during index creation reduces the insertion problem (typically $\approx 20\,\%$ free space).
- Since ISAM indexes are static, pages need not be **locked** (database jargon: "latched") during index access.
  - Latching can be a serious bottleneck in dynamic tree indexes (particularly near the root node).
- ISAM may be the index of choice for relatively static data.

B^+-trees: A Dynamic Index Structure

The **B^+-tree** is derived from the ISAM index, but is fully dynamic with respect to updates.

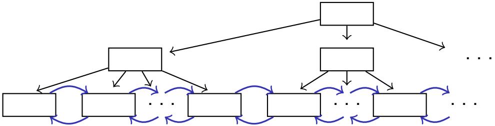
- **No overflow chains**; B^+-trees remain **balanced** at all times
- Gracefully adjusts to **inserts** and **deletes**.
- **Minimum occupancy** for all B^+-tree nodes (except the root): **50 %** (typically: 67 %).

- Original version: **B-tree**: R. Bayer and E. M. McCreight. Organization and Maintenance of Large Ordered Indexes. *Acta Informatica*, vol. 1, no. 3, September 1972.

B$^+$-trees: Basics

B$^+$-trees look like ISAM indexes, where

- leaf nodes are, generally, **not** in sequential order on disk,
- leaves are connected to form a **double-linked list**:[2]



- leaves may contain **actual data** (like the ISAM index) or just **references** to data pages (*e.g.*, rids). ↗ slides 79 and 85
  - We assume the **latter** case in the following, since it is the more common one.
- each B$^+$-tree node contains between d and $2d$ entries (d is the **order** of the B$^+$-tree; the root is the only exception)

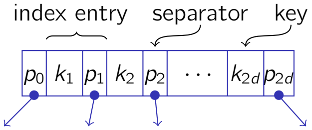
---

[2] This is not really a B$^+$-tree requirement, but some systems implement it.

```
Function: search (k)
return tree_search (k, root);
```

```
Function: tree_search (k, node)
if node is a leaf then
    return node;
switch k do
    case k < k_1 do
        return
         tree_search (k, p_0);
    case k_i ≤ k < k_{i+1} do
        return
         tree_search (k, p_i);
    case k_{2d} ≤ k do
        return
         tree_search (k, p_{2d});
```

- Function `search` (k) returns a pointer to the leaf node that contains potential hits for search key k.



node page layout

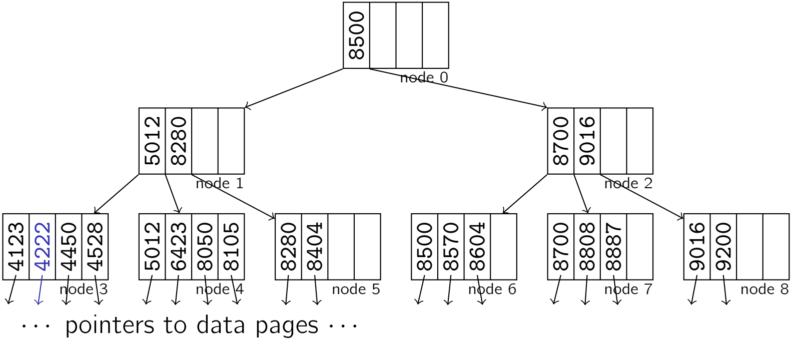
Insert: Overview

- The B^+-tree needs to remain **balanced** after every update.[3]
  $\rightarrow$ We **cannot** create overflow pages.
- Sketch of the insertion procedure for entry $\langle k, p \rangle$ (key value k pointing to data page p):
  1. **Find leaf page** n where we would expect the entry for k.
  2. If n has **enough space** to hold the new entry (*i.e.*, at most $2d - 1$ entries in n), **simply insert** $\langle k, p \rangle$ into n.
  3. Otherwise node n must be **split** into n and n' and a new **separator** has to be inserted into the parent of n.

     Splitting happens recursively and may eventually lead to a split of the root node (increasing the tree height).

[3] *I.e.*, every root-to-leaf path must have the same length.

Insert: Examples (Insert without Split)

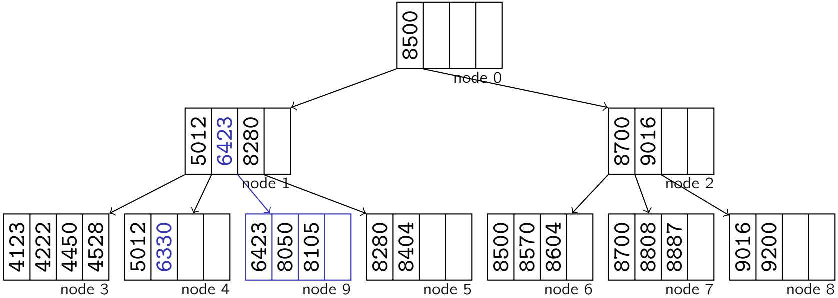


Insert new entry with key 4222.

→ Enough space in node 3, simply insert.

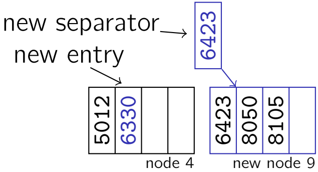
→ Keep entries **sorted within nodes**.

Insert: Examples (Insert with Leaf Split)

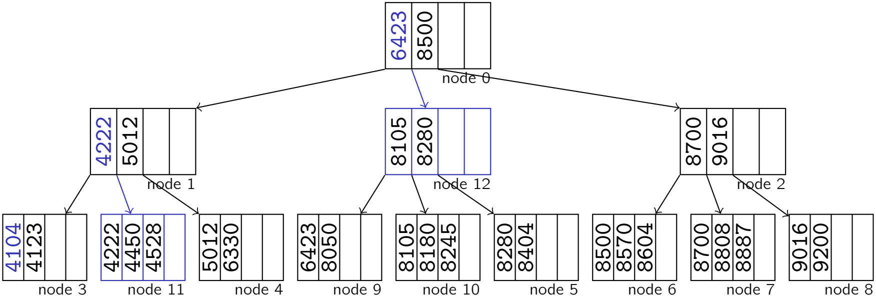


Insert key 6330.

- → Must **split** node 4.
- → **New separator** goes into node 1 (including pointer to new page).



Insert: Examples (Insert with Inner Node Split)

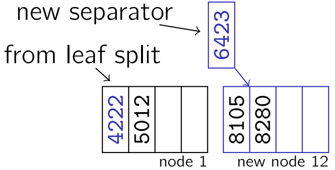


After 8180, 8245, insert key 4104.

→ Must **split** node 3.

→ Node 1 overflows → split it

→ **New separator** goes into root



Unlike during leaf split, separator key does **not** remain in inner node. ✎ **Why?**

Insert: Root Node Split

- Splitting starts at the leaf level and continues upward as long as index nodes are fully occupied.
- Eventually, this can lead to a split of the **root node**:
  - Split like any other inner node.
  - Use the separator to create a **new root**.
- The root node is the **only** node that may have an occupancy of less than 50 %.
- This is the **only** situation where the tree height increases.

✎ **How often do you expect a root split to happen?**

Insertion Algorithm

```
Function: tree_insert (k, rid, node)
if node is a leaf then
    return leaf_insert (k, rid, node);
else
    switch k do
        case k < k_1 do
            ⟨sep, ptr⟩ ← tree_insert (k, rid, p_0);
        case k_i ≤ k < k_{i+1} do
            ⟨sep, ptr⟩ ← tree_insert (k, rid, p_i);
        case k_2d ≤ k do
            ⟨sep, ptr⟩ ← tree_insert (k, rid, p_2d);
    if sep is null then
        return ⟨null, null⟩;
    else
        return split (sep, ptr, node);
```

see `tree_search ()`

Function: `leaf_insert` (k, rid, $node$)

if another entry fits into *node* **then**
  insert $\langle k, rid \rangle$ into *node* ;
  return $\langle \mathbf{null}, \mathbf{null} \rangle$;
else
  allocate new leaf page p ;
  take $\left\{ \langle k_1^+, p_1^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle \right\}$:= entries from *node* $\cup \{ \langle k, ptr \rangle \}$
    leave entries $\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_d^+, p_d^+ \rangle$ in *node* ;
    move entries $\langle k_{d+1}^+, p_{d+1}^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle$ to p ;
  return $\langle k_{d+1}^+, p \rangle$;

Function: `split` (k, ptr, $node$)

if another entry fits into *node* **then**
  insert $\langle k, ptr \rangle$ into *node* ;
  return $\langle \mathbf{null}, \mathbf{null} \rangle$;
else
  allocate new leaf page p ;
  take $\left\{ \langle k_1^+, p_1^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle \right\}$:= entries from *node* $\cup \{ \langle k, ptr \rangle \}$
    leave entries $\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_d^+, p_d^+ \rangle$ in *node* ;
    move entries $\langle k_{d+2}^+, p_{d+2}^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle$ to p ;
    set $p_0 \leftarrow p_{d+1}^+$ in p;
  return $\langle k_{d+1}^+, p \rangle$;

Insertion Algorithm

Function: `insert` (k, rid)

$\langle key, ptr \rangle \leftarrow$ `tree_insert` (k, rid, $root$);
if key is not **null then**
    allocate new root page r;
    populate r **with**
        $p_0 \leftarrow root$;
        $k_1 \leftarrow key$;
        $p_1 \leftarrow ptr$;
    $root \leftarrow r$;

- `insert` (k, rid) is called from outside.
- Note how leaf node entries point to rids, while inner nodes contain pointers to other B$^+$-tree nodes.

Index Pages

B-trees use slotted pages, too.

Inner Nodes:

- record $\equiv \langle key, childPage \rangle$ pairs.
- Additional key value to hold extra child pointer
  - *e.g.*, key value from reference in parent
  - "dummy key" for far-left or far-right end
- Similar to leaves, $\langle key, childPage\text{-}list \rangle$ might make sense, too.

Index Pages

Leaf Nodes: Three options:

1. Store full data records in B-tree leaf
   - $\rightarrow$ B-tree becomes a method to physically organize the table's data pages.
   - $\rightarrow$ "clustered index" or "index-organized table"
2. record $\equiv \langle key, rid\text{-}list \rangle$
   - $\rightarrow$ There could be more than one tuple for same key.
3. record $\equiv \langle key, rid \rangle$
   - $\rightarrow$ Easier when keys are unique. ✎ **Why?**

Options 2 and 3 are reasons why we want record ids to be **stable**.

$\rightarrow$ slides 50 ff.

Index Pages — IBM DB2

E.g., index on `VARCHAR` field with random content:

```
Hi Key 0:
   Offset Location = 668  (x29C)
   Record Length   = 455  (x1C7)
   Key Part 1:
       Variable Length Character String
       Actual Length = 0
   Child Pointer  =>  Page 24694
   Table RID: x(0000 03C6 0027) r(000003C6;0027) d(966;39)
   Child Pointer  =>  Page 24695
   Table RID: x(0000 0514 0018) r(00000514;0018) d(1300;24)
   ...
Hi Key 1:
   Offset Location = 1123  (x463)
   Record Length   = 31  (x1F)
   Key Part 1:
       Variable Length Character String
       Actual Length = 16
             2B2B357A 5169792F 31307556 73513D3D    ++5zQiy/10uVsQ==
   Child Pointer  =>  Page 24739
   Table RID: x(FFFF FFFF FFFF) r(FFFFFFFF;FFFF) d(4294967295;65535)
```

Slotted Pages for Data and Indexes

Data Pages:

- Move record without changing its slot/RID.

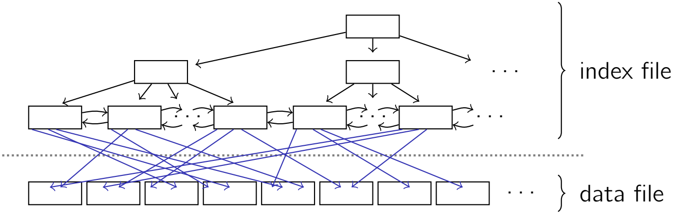
Index Pages:

- Also: change slots without moving data.

✎ **Huh?**

B^+-trees and Sorting

A typical situation according to alternative 2 looks like this:

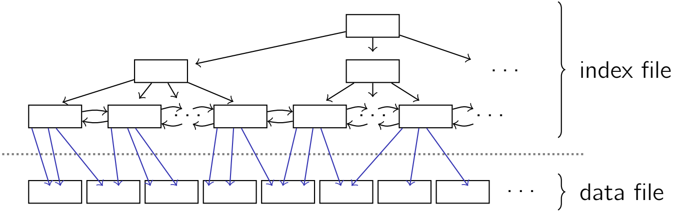


✎ **What are the implications when we want to execute**

`SELECT * FROM CUSTOMERS ORDER BY ZIPCODE` **?**

Clustered B$^+$-trees

If the data file was **sorted**, the scenario would look different:



We call such an index a **clustered index**.

- Scanning the index now leads to **sequential access**.
- This is particularly good for **range queries**.

✎ **Why don't we make all indexes clustered?**

Indexes and Clustering in IBM DB2

DB2 does **not** offer clustered indexes in the sense discussed here.

But:

- Can declare one "clustering index" per table.

```
CREATE INDEX IndexName
    ON TableName ( col1 , col2 , ... , coln ) CLUSTER
```

- DB2 will attempt (!) to cluster the table's **data pages** according to the key of the index.
  - Table **re-organization** will re-establish clustering if necessary.
  - Use `ALTER TABLE` and `PCTFREE` to ease future inserts.

Index Organized Tables

Alternative 1 (slide 79) is a special case of a clustered index.

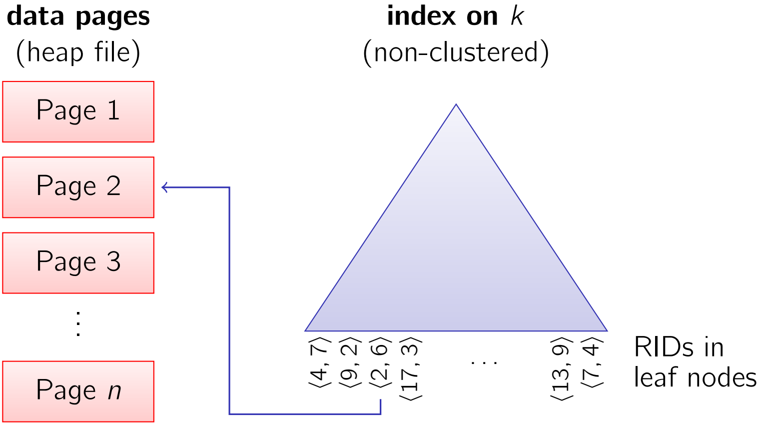
- index file ≡ data file
- Such a file is often called an **index organized table**.

E.g., Oracle8i

```
CREATE TABLE (...
              ...,
              PRIMARY KEY ( ... ))
  ORGANIZATION INDEX;
```

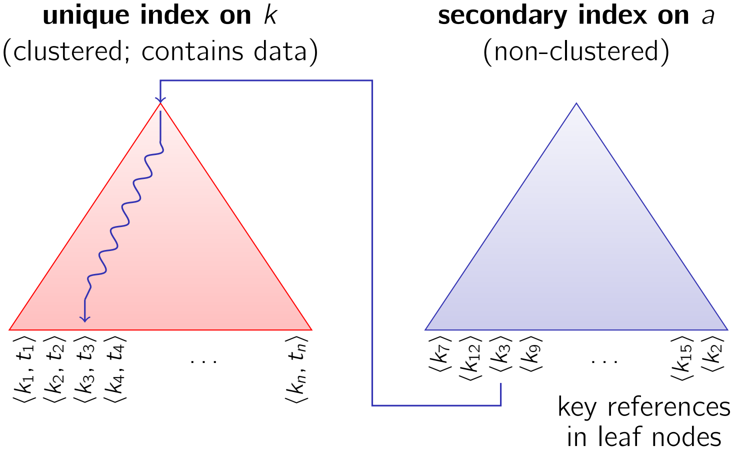
Indexes on Tables

Option A: Heap file for data, indexes with RIDs



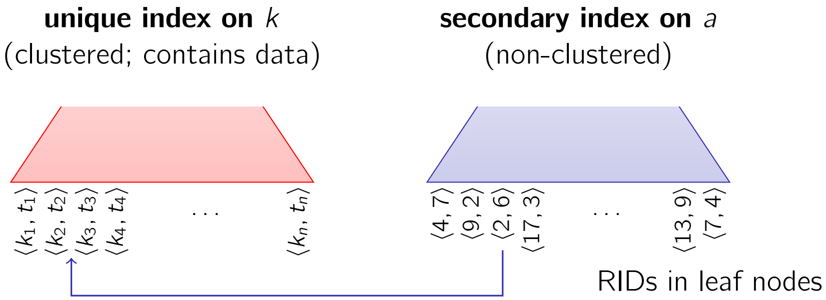
→ Can have arbitrarily many indexes of this kind.

Option B: Data sits in clustered index



- Secondary indexes use key values to reference tuples.

✎ **What about this setup?**



Prefix Truncation

Address book of Berlin, anno 1858:[4]

Poplawsky — Präger. 353

Poplawsky, C., Drechsler in Horn und Holz, Schornsteinfegergasse 1.
Poplowsky, W., Uhrmacher, Leipzigerstraße 43.
Popowitsch, R., Kürschner, Ziegelstraße 11. 12.
Popowska, M., Haushofmeisterfrau, Kochstr. 43.
Popp, C., Schlosser, Schießgasse 7.
— M., Schneider f. D., Kommandantenstr. 31.
— A. C., Seidenwirker, Prinzen-Allee 74.
— F., geb. Schmidt, Ww., Gutsbesitzerin, Kronenstr. 17.
Poppe, C. T., Diätar, J., Wilhelmsstraße 92.
— W., Cafetier, Schustergasse 1.
— E., Holz- und Horndrechsler, Wallstraße 54.
— Eduard, Filz- und Filzschuhfabrik, Wolldruckerei u. Lager von Filzschuh-Oberstoff, Plüschband und aller Arten Filzwaaren, Friedrichsstr. 109. E.
— W., Fuhrherr, Neu-Kölln a. W. 21.
— J., Handelsmann, Sophienstr. 26.

Porath, H., Oberfeuermann, Landwehrstr. 3.
— W., Gärtner und Blumenhändler, Oranienburgerstr. 57.
— F., Tuchmacher, Weberstr. 34.
Parawsky, S., Eisenbahn-Zugführer, Louisenplatz 12.
Porepp, C., Zimmer-Vermiether, Unter den Linden 47.
Pormetter, F. W., Buchdruckereibesitzer, Kommandantenstr. 7.
Porsch, H., Kunstmaler, Heiligegeiststraße 25.
Porschien, G., Schneider, Friedrichsgracht 59.
Port, C., Tapezirer, Fruchtstr. 58.
Portesset, J., Polizei-Wachtmeister, Charlottenstr. 37.
— M. u. J., Geschw., Putz- und Modehändlerinnen, Charlottenstr. 37.
Porth, C., Hofschauspieler, Friedrichsstraße 195.
— H., Tischler, Markgrafenstr. 18.
Porthum, G., Klempner, Rosenstr. 8.
Portier, L., Dlle., Lindenstr. 48.

Possart, F., Intendantur-Applicant, Schumannsstr. 9. 2—4.
— E., Kaufmann, Inhaber des landwirthschaftlichen Etablissements, Heiligegeiststr. 3. F. Eugen Possart. Cp.
— J. C., Kaufmann, Schumannsstr. 9. 12—2.
— F., Schankwirth, Köpnickerstr. 129.
Posse, C., Barbier, Mauerstr. 33.
— L., Schneider f. H., Neuer Markt 9.
— A., Schneider f. H., Krausenstr. 4. 5.
Possel, F., Handelsmann, Dresdnerstraße 97.
— F., Musikus, Lindenstr. 56.
Posseldt, J., Geh. Registratur-Assistent im Ministerium für Handel rc., Grabenstr. 3. 4—5.
Posselt, P., Kanzleidiener, Leipzigerstraße 5.
— C., Modelleur, Stralauerplatz 4.
C., Porzellanmaler, Alte Jakobsstraße 60. E.
— L., Koch und Restaurateur, Charitéstraße 5.
— L., Restaurateur, Mittelstr. 57.

[4] http://adressbuch.zlb.de/

Prefix Truncation

Address book:

- To save space, common last names are printed only once.

Such **prefix truncation** can also be applied to B-trees:[5]

Prefix: `Smith, J`								
`ack`	`ane`	`ason`	`eremy`	`ill`	`ohn`	. . .	`une`	

The advantage is **two-fold**:

1. save space → more keys fit on one page → higher fanout
2. need **fewer comparisons**

[5] R. Bayer and K. Unterauer. *Prefix B-Trees*. TODS 2(1), March 1972.

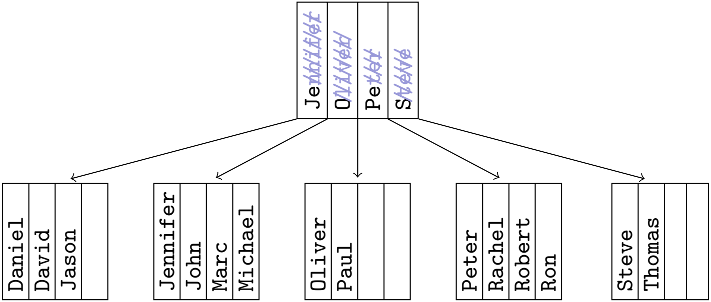
Suffix Truncation

- **Prefix truncation** is most effective near or in leaf pages. ✎ **Why?**
- Elsewhere, by contrast, the leading key parts are most discriminative.
- In fact, a key's suffix might not be needed to guide navigation at all.

This motivates **suffix truncation**:

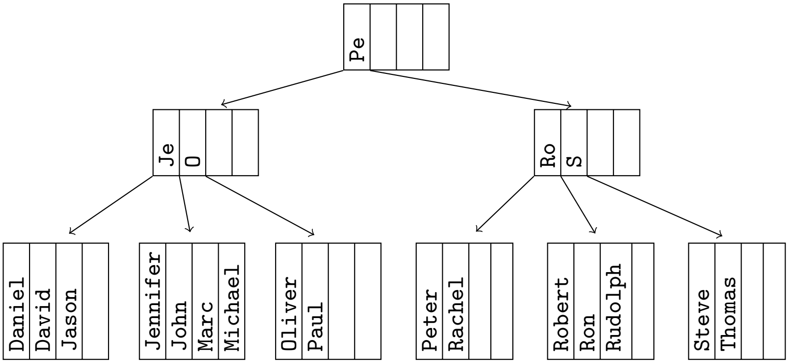
- Store keys only as far as needed to guide search.
- Remember: key values in inner tree nodes do **not** have to be contained in the actual data set.

Example:


Je~~nnifer~~
O~~liver~~
Pe~~ter~~
S~~teve~~
Daniel
David
Jason
Jennifer
John
Marc
Michael
Oliver
Paul
Peter
Rachel
Robert
Ron
Steve
Thomas

Suffix Truncation

Suffix truncation beyond the bottom-most level is difficult/dangerous.

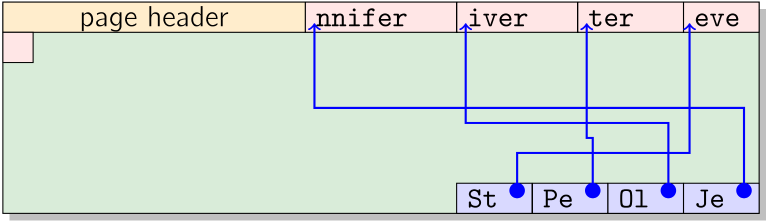


→ Shortening '`Pe`' to '`P`' would be incorrect!

"Poor Man's Normalized Keys"

The effect of discriminative prefixes can also exploited as follows:

- Store a **fixed-length prefix** as an additional field in every entry of the slot directory.
- Need to follow the pointer only if the prefix is not enough to decide on the comparison outcome.



"Poor Man's Normalized Keys"

- Most accesses are to an array of fixed-length elements (Pointer chasing in memory is relatively expensive on modern hardware.)
- Can use, *e.g.*, integer comparisons to evaluate four-byte prefix comparisons.

  May need to **re-order** bytes for this.
- **CPU cache efficient:** When a slot entry is read, likely the prefix is in the same cache line.

Key Normalization

In practice, key comparisons are not as simple as they look on slides:

- language-specific **collation**
- representations as different **character sets**
- `NULL` values

Plus, keys might be composed of **multiple columns**.

Thus:

- **Normalize** keys and represent any key as a **bit string**.
  - → All of the above issues only affect normalization, but not B-tree operations themselves.
- Can prepare, *e.g.*, for integer (rather than bit or byte) comparisons.

Key Normalization

Examples:

- Map upper and lower case letters to same bit string if collation is case insensitive.
- Use bit representations for characters according to collation *E.g.*, ö < `z` in German; `z` < ö in Swedish.
- To sort `NULL` before any value: Prepend any valid value with a '1' bit and represent `NULL` as a '0' bit.

A	*B*	*C*	normalized key
2	'`Smith`'	'`John`'	$\underline{1}$ 00···00000010 $\underline{1}$ `Smith`'\0' $\underline{1}$ `John`'\0'
3	'`Miller`'	''	$\underline{1}$ 00···00000011 $\underline{1}$ `Miller`'\0' $\underline{1}$ '\0'
64	–	'`Dave`'	$\underline{1}$ 00···00100000 $\underline{0}$ $\underline{1}$ `Dave`'\0'
–	''	–	$\underline{0}$ $\underline{1}$ '\0' $\underline{0}$

Key Normalization

Information might get lost during normalization (*e.g.*, capitalization)

→ Store normalized **and** original key (redundantly) in leaf nodes or

→ Use normalization only in inner nodes

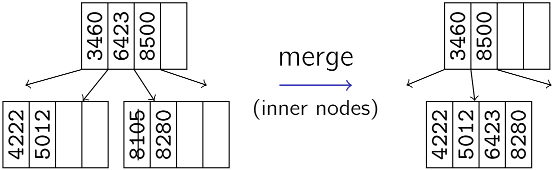
Keys tend to become larger due to normalization.

→ **Order-preserving compression** might be useful.

Key normalization and prefix/suffix truncation go particularly well together.

Deletion

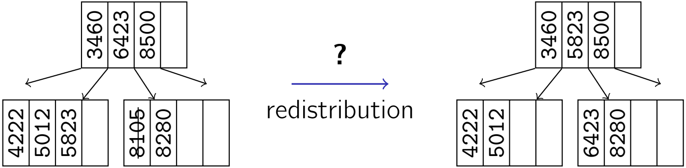
- If a node is sufficiently full (*i.e.*, contains at least $d + 1$ entries, we may simply remove the entry from the node.
  - Note: Afterward, **inner nodes** may contain keys that no longer exist in the database. This is perfectly legal.
- **Merge** nodes in case of an **underflow** ("undo a split"):



  - "Pull" separator into merged node.

Deletion

It's not quite that easy...



- Merging only works if **two** neighboring nodes were 50 % full.
- Otherwise, we have to **re-distribute**:
  - "rotate" entry through parent

B^+-trees in Real Systems

- Actual systems often avoid the cost of merging and/or redistribution, but relax the minimum occupancy rule.
- To improve **concurrency**, systems sometimes only **mark** index entries as deleted and physically remove them later (*e.g.*, IBM DB2 UDB "type-2 indexes")
  - → **"Ghost bits" / "ghost records"**
  - → Often kept around for a while → re-use on next insert.

Ghost Records — IBM DB2

Before key deletion:

```
Key 64:
 Offset Location = 3710  (xE7E)
 Record Length   = 40  (x28)
 Key Part 1:
     Variable Length Character String
     Actual Length = 28
           44516A6B 7A334650 76724471 534B7767    DQjkz3FPvrDqSKwg
     58432B59 345A7837 4852383D             XC+Y4Zx7HR8=
 Table RID: x(0000 1237 0001) r(00001237;0001) d(4663;1) ridFlags=x0
```

After key deletion:

```
Key 64:
 Offset Location = 3710  (xE7E)
 Record Length   = 40  (x28)
 Key Part 1:
     Variable Length Character String
     Actual Length = 28
           44516A6B 7A334650 76724471 534B7767    DQjkz3FPvrDqSKwg
     58432B59 345A7837 4852383D             XC+Y4Zx7HR8=
 Table RID: x(0000 1237 0001) r(00001237;0001) d(4663;1) ridFlags=x3 Punc Deleted
```

Key Deletion — IBM DB2

In IBM DB2, redistribution and merging are **only** applied if

- the page is a **leaf node** and
  (Remember the pointers between adjacent leaf nodes, ↗ slide 68.)
- the fill degree of the page falls below `MINPCTUSED` and
  (That also means that `MINPCTUSED` must have a value greater than its default, which is 0.)
- the transaction holds an **exclusive lock on the table**.

This is called **online index defragmentation** in DB2.

Otherwise, “clean-up” only happens during explicit index maintenance.

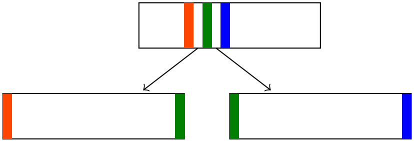
- Use `REORG INDEX` to trigger maintenance.
- Use `REORGCHK` to check whether index(es) need maintenance.

Ghost Records

Ghost records turn out to be useful for a number of purposes.

E.g., **fence keys**

- Keep a copy of parent's separator keys in every node



- Fence keys span range of **possible** key values in this node
  → Avoids problems with **prefix truncation**.
- One key is an **exclusive bound**, thus **must** be a ghost record.
- The other one may or may not be a ghost record.
- Can be used, *e.g.*, to check **integrity** of B-tree.

Variable-Length Keys

With **variable-length keys**, the original B-tree property

$$d \leq \textit{number of keys in a node} \leq 2d$$

is not practical any more.

→ Real-world systems do not really care about this "50 % rule."

With truncation, the storage space for a key might even **change** during re-organizations.

- ✎ Will this cause any trouble during updates?

Composite Keys

B$^+$-trees can (in theory[6]) be used to index everything with a defined **total order**, *e.g.*:

- integers, strings, dates, . . . , and
- **concatenations** thereof (based on **lexicographical order**).

E.g., in most SQL dialects:

```
CREATE INDEX ON TABLE CUSTOMERS (LASTNAME, FIRSTNAME);
```

A useful application are, *e.g.*, **partitioned B-trees**:

- Leading index attributes effectively **partition** the resulting B$^+$-tree.

↗ G. Graefe: Sorting And Indexing With Partitioned B-Trees. *CIDR 2003*.

[6]Some implementations won't allow you to index, *e.g.*, large character fields.

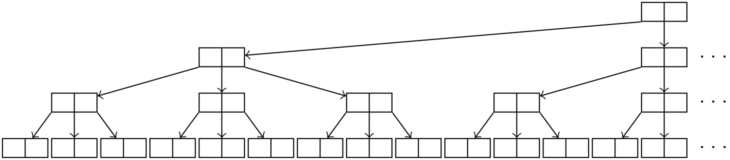
Partitioned B-trees

```
CREATE INDEX ON TABLE STUDENTS (SEMESTER, ZIPCODE);
```

✎ **What types of queries could this index support?**

Bulk-Loading B^+-trees

Building a B^+-tree is particularly easy when the input is **sorted**.



- Build B^+-tree **bottom-up** and **left-to-right**.
- Create a parent for every $2d + 1$ unparented nodes.
  (Actual implementations typically leave some space for future updates.
  ↗ *e.g.*, DB2's `PCTFREE` parameter)

✎ **What use cases could you think of for bulk-loading?**

Stars, Pluses, . . .

In the foregoing we described the **B$^+$-tree**.

Bayer and McCreight originally proposed the **B-tree**:

- Inner nodes contain data entries, too. ✎ **Pros/cons?**

There is also a **B*-tree**:

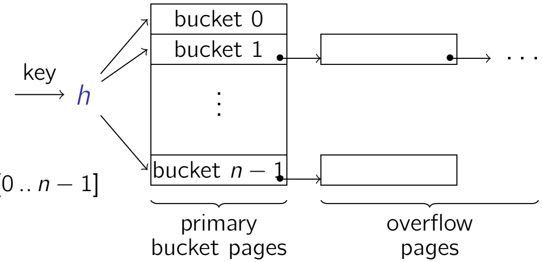
- Keep non-root nodes at least 2/3 full (instead of 1/2).
- Need to **redistribute on inserts** to achieve this.
  (Whenever **two** nodes are full, split them into **three**.)

Most people say "B-tree" and mean any of these variations. Real systems typically implement B$^+$-trees.

"B-trees" are also used outside the database domain, *e.g.*, in modern **file systems** (ReiserFS, HFS, NTFS, . . .).

Hash-Based Indexing

B$^+$-trees are **by far** the predominant type of indices in databases. An alternative is **hash-based indexing**.



$h : \mathrm{dom}(key) \rightarrow [0 .. n-1]$

- Hash indices can only be used to answer **equality predicates**.
- Particularly good for strings (even for very long ones).

Dynamic Hashing

Problem: How do we choose n (the number of buckets)?

- n too large → space wasted, poor space locality
- n too small → many overflow pages, degrades to linked list

Database systems, therefore, use **dynamic hashing** techniques:

- **extendible hashing**,
- **linear hashing**.

Few systems support true hash indices (*e.g.*, PostgreSQL).

More popular uses of hashing are:

- support for **B$^+$-trees** over hash values (*e.g.*, SQL Server)
- the use of hashing during query processing → **hash join**.

Recap

Indexed Sequential Access Method (ISAM)

A **static**, tree-based index structure.

B^+-trees

The database index structure; indexing based on any kind of (linear) **order**; adapts **dynamically** to inserts and deletes; low tree heights ($\sim$ 3–4) guarantee fast lookups.

Clustered vs. Unclustered Indices

An index is clustered if its underlying data pages are ordered according to the index; fast **sequential access** for clustered B^+-trees.

Hash-Based Indices

Extendible hashing and **linear hashing** adapt dynamically to the number of data entries.