Part III

Indexing
How could we prepare for such queries and evaluate them efficiently?

We could

1. sort the table on disk (in ZIPCODE order).
2. To answer queries, then use binary search to find first qualifying tuple, and scan as long as ZIPCODE < 8999.

\[ \text{k* denotes the full data record with search key } k. \]
We get **sequential access** during the **scan phase**.

We need to read $\log_2(\# \text{ tuples})$ tuples during the **search phase**.

We need to read about as many **pages** for this.

(The whole point of binary search is that we make far, unpredictable jumps. This largely defeats prefetching.)
Observations:

- Make rather **far jumps initially**.
  - For each step read **full page**, but inspect only **one record**.
- After $O(\log_2 \text{pagesize})$, search stays **within one page**.
  - I/O cost is used much more efficiently here.
**Idea:** “Cache” those records that might be needed for the first phase.

→ **If we can keep the cache in memory, we can find any record with just a single I/O.**

🔍 **Is this assumption reasonable?**
What if my data set is really large?

- “Cache” will span many pages, too.
  (In practice, we'll organize the cache just like any other database object.)
- Thus: “cache the cache” → hierarchical “cache”
**Idea:** Accelerate the search phase using an index.

- All nodes are the size of a page
  - hundreds of entries per page
  - large fanout, low depth
- Search effort: $\log_{\text{fanout}}(\#\text{ tuples})$
ISAM Index: Updates

ISAM indexes are inherently **static**.

- **Deletion** is not a problem: delete record from data page.
- **Inserting** data can cause more effort:
  - If space is left on respective leaf page, insert record there (e.g., after a preceding deletion).
  - Otherwise, **overflow pages** need to be added. (Note that these will **violate** the sequential order.)
- ISAM indexes **degrade** after some time.
Remarks

- Leaving some free space during index creation reduces the insertion problem (typically \( \approx 20\% \) free space).
- Since ISAM indexes are static, pages need not be locked (database jargon: “latched”) during index access.
  - Latching can be a serious bottleneck in dynamic tree indexes (particularly near the root node).
- ISAM may be the index of choice for relatively static data.
The **B⁺-tree** is derived from the ISAM index, but is fully dynamic with respect to updates.

- **No overflow chains**; B⁺-trees remain **balanced** at all times
- Gracefully adjusts to **inserts** and **deletes**.
- **Minimum occupancy** for all B⁺-tree nodes (except the root): **50%** (typically: 67%).

B⁺-trees: Basics

B⁺-trees look like ISAM indexes, where

- leaf nodes are, generally, not in sequential order on disk,
- leaves are connected to form a double-linked list:\(^2\)
- leaves may contain actual data (like the ISAM index) or just references to data pages (e.g., rids). We assume the latter case in the following, since it is the more common one.
- each B⁺-tree node contains between \(d\) and \(2d\) entries (\(d\) is the order of the B⁺-tree; the root is the only exception)

\(^2\)This is not really a B⁺-tree requirement, but some systems implement it.
Searching a B$^+$-tree

Function: search($k$)

return tree_search($k$, root);

Function: tree_search($k$, node)

if node is a leaf then
  return node;

switch $k$ do
  case $k < k_1$ do
    return tree_search($k$, $p_0$);
  case $k_i \leq k < k_{i+1}$ do
    return tree_search($k$, $p_i$);
  case $k_{2d} \leq k$ do
    return tree_search($k$, $p_{2d}$);

Function search($k$) returns a pointer to the leaf node that contains potential hits for search key $k$. 
Insert: Overview

- The B\(^+\)-tree needs to remain balanced after every update.\(^3\)
  - We cannot create overflow pages.
- Sketch of the insertion procedure for entry \(\langle k, p \rangle\)
  (key value \(k\) pointing to data page \(p\)):
  1. Find leaf page \(n\) where we would expect the entry for \(k\).
  2. If \(n\) has enough space to hold the new entry (\(i.e.,\) at most \(2d - 1\) entries in \(n\)), simply insert \(\langle k, p \rangle\) into \(n\).
  3. Otherwise node \(n\) must be split into \(n\) and \(n'\) and a new separator has to be inserted into the parent of \(n\).

Splitting happens recursively and may eventually lead to a split of the root node (increasing the tree height).

---

\(^3\)\(i.e.,\) every root-to-leaf path must have the same length.
Insert new entry with key 4222.

→ Enough space in node 3, simply insert.

→ Keep entries sorted within nodes.
Insert: Examples (Insert with Leaf Split)

Insert key 6330.

→ Must **split** node 4.

→ **New separator** goes into node 1 (including pointer to new page).
After 8180, 8245, insert key 4104.

→ Must split node 3.
→ Node 1 overflows → split it
→ New separator goes into root

Unlike during leaf split, separator key does not remain in inner node.  🤔 Why?
Splitting starts at the leaf level and continues upward as long as index nodes are fully occupied.

Eventually, this can lead to a split of the root node:
- Split like any other inner node.
- Use the separator to create a new root.

The root node is the **only** node that may have an occupancy of less than 50%.

This is the **only** situation where the tree height increases.

**How often do you expect a root split to happen?**
Insertion Algorithm

1 Function: tree_insert(k, rid, node)
2 if node is a leaf then
3 return leaf_insert(k, rid, node);
4 else
5 switch k do
6 case k < k₁ do
7 ⟨sep, ptr⟩ ← tree_insert(k, rid, p₀);
8 case kᵢ ≤ k < kᵢ₊₁ do
9 ⟨sep, ptr⟩ ← tree_insert(k, rid, pᵢ);
10 case k₂ᵈ ≤ k do
11 ⟨sep, ptr⟩ ← tree_insert(k, rid, p₂ᵈ);
12 if sep is null then
13 return ⟨null, null⟩;
14 else
15 return split(sep, ptr, node);

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Function: leaf_insert ($k$, $rid$, node)

if another entry fits into node then
    insert $\langle k, rid \rangle$ into node;
    return $\langle \text{null}, \text{null} \rangle$;
else
    allocate new leaf page $p$;
    take $\{\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle\} :=$ entries from node $\cup \{\langle k, ptr \rangle\}$
    leave entries $\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_d^+, p_d^+ \rangle$ in node;
    move entries $\langle k_{d+1}^+, p_{d+1}^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle$ to $p$;
    return $\langle k_{d+1}^+, p \rangle$.

Function: split ($k$, $ptr$, node)

if another entry fits into node then
    insert $\langle k, ptr \rangle$ into node;
    return $\langle \text{null}, \text{null} \rangle$;
else
    allocate new leaf page $p$;
    take $\{\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle\} :=$ entries from node $\cup \{\langle k, ptr \rangle\}$
    leave entries $\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_d^+, p_d^+ \rangle$ in node;
    move entries $\langle k_{d+2}^+, p_{d+2}^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle$ to $p$;
    set $p_0 \leftarrow p_{d+1}^+$ in node;
    return $\langle k_{d+1}^+, p \rangle$;
Insertion Algorithm

1. **Function**: `insert (k, rid)`
2. \( \langle key, ptr \rangle \leftarrow \text{tree\_insert} (k, rid, root); \)
3. **if** `key` is not **null** **then**
4. allocate new root page \( r; \)
5. **populate** \( r \) **with**
6. \( p_0 \leftarrow root; \)
7. \( k_1 \leftarrow key; \)
8. \( p_1 \leftarrow ptr; \)
9. \( root \leftarrow r; \)

- `insert (k, rid)` is called from outside.
- Note how leaf node entries point to rids, while inner nodes contain pointers to other \( B^+ \)-tree nodes.
B-trees use slotted pages, too.

**Inner Nodes:**

- record $\equiv \langle \text{key}, \text{childPage} \rangle$ pairs.
- Additional key value to hold extra child pointer
  - e.g., key value from reference in parent
  - “dummy key” for far-left or far-right end
- Similar to leaves, $\langle \text{key}, \text{childPage-list} \rangle$ might make sense, too.
**Leaf Nodes:** Three options:

1. Store full data records in B-tree leaf
   - B-tree becomes a method to physically organize the table’s data pages.
   - “clustered index” or “index-organized table”

2. record ≡ \langle key, rid-list \rangle
   - There could be more than one tuple for same key.

3. record ≡ \langle key, rid \rangle
   - Easier when keys are unique. ✉️ **Why?**

Options 2 and 3 are reasons why we want record ids to be **stable**.
- slides 51 ff.
E.g., index on VARCHAR field with random content:

Hi Key 0:
    Offset Location = 668 (x29C)
    Record Length = 455 (x1C7)
    Key Part 1:
        Variable Length Character String
        Actual Length = 0
    Child Pointer => Page 24694
    Table RID: x(0000 03C6 0027) r(000003C6;0027) d(966;39)
    Child Pointer => Page 24695
    Table RID: x(0000 0514 0018) r(00000514;0018) d(1300;24)
    ...

Hi Key 1:
    Offset Location = 1123 (x463)
    Record Length = 31 (x1F)
    Key Part 1:
        Variable Length Character String
        Actual Length = 16
        2B2B357A 5169792F 31307556 73513D3D ++5zQiy/10uVsQ==
    Child Pointer => Page 24739
    Table RID: x(FFFF FFFF FFFF) r(FFFFFFFF;FFFF) d(4294967295;65535)
Data Pages:
- Move record without changing its slot/RID.

Index Pages:
- Also: change slots without moving data.

✍ Huh?
A typical situation according to alternative 2 looks like this:

What are the implications when we want to execute

```sql
SELECT * FROM CUSTOMERS ORDER BY ZIPCODE
```
If the data file was **sorted**, the scenario would look different:

We call such an index a **clustered index**.

- Scanning the index now leads to **sequential access**.
- This is particularly good for **range queries**.

Why don’t we make all indexes clustered?
Indexes and Clustering in IBM DB2

DB2 does **not** offer clustered indexes in the sense discussed here.

But:

- Can declare one “clustering index” per table.

```sql
CREATE INDEX IndexName
ON TableName (col1, col2, ..., col_n) CLUSTER
```

- DB2 will attempt (!) to cluster the table’s **data pages** according to the key of the index.
  - Table **re-organization** will re-establish clustering if necessary.
  - Use `ALTER TABLE` and `PCTFREE` to ease future inserts.
Index Organized Tables

Alternative 1 (slide 80) is a special case of a clustered index.

- index file ≡ data file
- Such a file is often called an **index organized table**.

E.g., Oracle8i

```sql
CREATE TABLE (... 
    ..., 
    PRIMARY KEY ( ... ))

ORGANIZATION INDEX;
```
Option A: Heap file for data, indexes with RIDs

- data pages (heap file)
  - Page 1
  - Page 2
  - Page 3
  - ... Page n

- index on $k$ (non-clustered)
  - RIDs in leaf nodes

→ Can have arbitrarily many indexes of this kind.
Indexes on Tables

Option B: Data sits in clustered index

unique index on $k$
(clustered; contains data)

secondary index on $a$
(non-clustered)

- Secondary indexes use key values to reference tuples.
What about this setup?

**unique index on** $k$
(clustered; contains data)

**secondary index on** $a$
(non-clustered)

RIDs in leaf nodes
# Address book of Berlin, anno 1858

## Poplawski — Prager

<table>
<thead>
<tr>
<th>Name</th>
<th>Occupation</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poplawski</td>
<td>C., Dreher in Holz und Holz, Schornsteinfegergasse 1.</td>
<td></td>
</tr>
<tr>
<td>Poplawski</td>
<td>B., Uhrmacher, Leipzigerstr. 43.</td>
<td></td>
</tr>
<tr>
<td>Popowitsch</td>
<td>N., Kürschner, Ziegelstr. 11, 12.</td>
<td></td>
</tr>
<tr>
<td>Popowska</td>
<td>N., Haushofmeisterin, Koschstr. 43.</td>
<td></td>
</tr>
<tr>
<td>Popp</td>
<td>C., Schlosser, Schiefagasse 7.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M., Schneider f. D., Kommandantenstr. 31.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A. C., Seidenwirker, Prinzen-Allee 74.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>J., geb. Schmidt, Mw., Gutsbesitzerin, Kronenstr. 17.</td>
<td></td>
</tr>
<tr>
<td>Poppe</td>
<td>C. E., Dietar, I., Wilhelmstr. 92.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M., Cafetier, Schusteragasse 1.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>E., Holz- und Hornbrechsler, Wallstr. 54.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Eduard, Filz- und Filzschuhfabrik, Boldruckerei u. Lager von Filzschuh-Deribo, Flussband und aller Arten Filzwaren, Friedrichstr. 109. E.</td>
<td></td>
</tr>
<tr>
<td>Porath</td>
<td>H., Oberschulmann, Landwehrstr. 3.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B., Gärtnerei und Blumenhändler, Dranenburgerstr. 57.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>J., Buchmacher, Weberstr. 34.</td>
<td></td>
</tr>
<tr>
<td>Parawski</td>
<td>G., Eisenbahn-Zugsführer, Louiseplatz 12.</td>
<td></td>
</tr>
<tr>
<td>Porepp</td>
<td>C., Zimmer-Vertrieb, Unter den Linden 47.</td>
<td></td>
</tr>
<tr>
<td>Pormetter</td>
<td>J. M., Buchdrucker, Kommandantenstr. 7.</td>
<td></td>
</tr>
<tr>
<td>Porst</td>
<td>H., Kunstgewerbe, Seilgießergasse 25.</td>
<td></td>
</tr>
<tr>
<td>Porschien</td>
<td>G., Schneider, Friedrichstr. 59.</td>
<td></td>
</tr>
<tr>
<td>Port</td>
<td>C., Tapiserie, Frühstr. 58.</td>
<td></td>
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<td>Porteset</td>
<td>J., Polizei- und Wachtmeister, Charlottenstr. 37.</td>
<td></td>
</tr>
<tr>
<td>Portz</td>
<td>E., Hofbuchmacher, Friedrichstr. 195.</td>
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<tr>
<td>Porth</td>
<td>H., Hofbauwirt, Friedrichstr. 195.</td>
<td></td>
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<tr>
<td>Porthum</td>
<td>G., Klempner, Rosenstr. 8.</td>
<td></td>
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<tr>
<td>Portier</td>
<td>E., Dilettante, Linienstr. 48.</td>
<td></td>
</tr>
<tr>
<td>Possart</td>
<td>J., Intendantur-Applicant, Schumannstr. 9. 2—4.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>J. E., Kaufmann, Schumannstr. 9. 12—2.</td>
<td></td>
</tr>
<tr>
<td>Pott</td>
<td>C., Schuhmacher, Königstr. 129.</td>
<td></td>
</tr>
</tbody>
</table>

---

4 [http://adressbuch.zlb.de/](http://adressbuch.zlb.de/)
Prefix Truncation

Address book:

- To save space, common last names are printed only once.

Such **prefix truncation** can also be applied to B-trees:\(^5\)

<table>
<thead>
<tr>
<th>Prefix: Smith, J</th>
</tr>
</thead>
<tbody>
<tr>
<td>ace</td>
</tr>
<tr>
<td>ane</td>
</tr>
<tr>
<td>ason</td>
</tr>
<tr>
<td>emery</td>
</tr>
<tr>
<td>ill</td>
</tr>
<tr>
<td>ohm</td>
</tr>
<tr>
<td>(\ldots)</td>
</tr>
<tr>
<td>une</td>
</tr>
</tbody>
</table>

The advantage is **two-fold**:

1. save space → more keys fit on one page → higher fanout
2. need **fewer comparisons**

---

Prefix truncation is most effective near or in leaf pages. Why?

Elsewhere, by contrast, the leading key parts are most discriminative.

In fact, a key’s suffix might not be needed to guide navigation at all.

This motivates suffix truncation:

- Store keys only as far as needed to guide search.
- Remember: key values in inner tree nodes do not have to be contained in the actual data set.
Suffix Truncation

Example:

- Jennifer
- Oliver
- Peter
- Steve
- Daniel
- Jennifer
- Oliver
- Peter
- Steve
- David
- John
- Paul
- Rachel
- Thomas
- Jason
- Marc
- Ron
- Steve
- Thomas
Suffix truncation beyond the bottom-most level is difficult/dangerous.

→ Shortening ‘Pe’ to ‘P’ would be incorrect!
The effect of discriminative prefixes can also be exploited as follows:

- Store a **fixed-length prefix** as an additional field in every entry of the slot directory.
- Need to follow the pointer only if the prefix is not enough to decide on the comparison outcome.
Most accesses are to an array of fixed-length elements (Pointer chasing in memory is relatively expensive on modern hardware.)

Can use, e.g., integer comparisons to evaluate four-byte prefix comparisons.

May need to re-order bytes for this.

CPU cache efficient: When a slot entry is read, likely the prefix is in the same cache line.
In practice, key comparisons are not as simple as they look on slides:

- language-specific **collation**
- representations as different **character sets**
- **NULL** values

Plus, keys might be composed of **multiple columns**.

**Thus:**

- **Normalize** keys and represent any key as a **bit string**.
  - All of the above issues only affect normalization, but not B-tree operations themselves.
- Can prepare, e.g., for integer (rather than bit or byte) comparisons.
Key Normalization

Examples:

- Map upper and lower case letters to same bit string if collation is case insensitive.

- Use bit representations for characters according to collation. *E.g.*, ö < z in German; z < ö in Swedish.

- To sort NULL before any value: Prepend any valid value with a ‘1’ bit and represent NULL as a ‘0’ bit.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>normalized key</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>‘Smith’</td>
<td>‘John’</td>
<td>100⋯00000010</td>
</tr>
<tr>
<td>3</td>
<td>‘Miller’</td>
<td>‘Dave’</td>
<td>100⋯00000011</td>
</tr>
<tr>
<td>64</td>
<td>–</td>
<td>–</td>
<td>100⋯00100000</td>
</tr>
</tbody>
</table>
Key Normalization

⚠️ Information might get lost during normalization (e.g., capitalization)

→ Store normalized **and** original key (redundantly) in leaf nodes or
→ Use normalization only in inner nodes

Keys tend to become larger due to normalization.

→ **Order-preserving compression** might be useful.

Key normalization and prefix/suffix truncation go particularly well together.
Deletion

- If a node is sufficiently full (i.e., contains at least $d+1$ entries, we may simply remove the entry from the node.
  - Note: Afterward, **inner nodes** may contain keys that no longer exist in the database. This is perfectly legal.

- **Merge** nodes in case of an **underflow** (“undo a split”):

  ![Diagram of merge](image)

  - “Pull” separator into merged node.
Deletion

It’s not quite that easy...

- Merging only works if **two** neighboring nodes were 50% full.
- Otherwise, we have to **re-distribute**: 
  - “rotate” entry through parent
Actual systems often avoid the cost of merging and/or redistribution, but relax the minimum occupancy rule.

To improve **concurrency**, systems sometimes only **mark** index entries as deleted and physically remove them later (e.g., IBM DB2 UDB “type-2 indexes”)

- “Ghost bits” / “ghost records”
- Often kept around for a while → re-use on next insert.
Before key deletion:

Key 64:

Offset Location = 3710 (xE7E)
Record Length = 40 (x28)

Key Part 1:
Variable Length Character String
Actual Length = 28
44516A6B 7A334650 76724471 534B7767 DQjkz3FPvrDqSKwg
58432B59 345A7837 4852383D XC+Y4Zx7HR8=
Table RID: x(0000 1237 0001) r(00001237;0001) d(4663;1) ridFlags=x0

After key deletion:

Key 64:

Offset Location = 3710 (xE7E)
Record Length = 40 (x28)

Key Part 1:
Variable Length Character String
Actual Length = 28
44516A6B 7A334650 76724471 534B7767 DQjkz3FPvrDqSKwg
58432B59 345A7837 4852383D XC+Y4Zx7HR8=
Table RID: x(0000 1237 0001) r(00001237;0001) d(4663;1) ridFlags=x3 Punc Deleted
In IBM DB2, redistribution and merging are only applied if

- the page is a leaf node and
  (Remember the pointers between adjacent leaf nodes, slide 69.)
- the fill degree of the page falls below MINPCTUSED and
  (That also means that MINPCTUSED must have a value greater than its default, which is 0.)
- the transaction holds an exclusive lock on the table.

This is called online index defragmentation in DB2.

Otherwise, “clean-up” only happens during explicit index maintenance.

- Use REORG INDEX to trigger maintenance.
- Use REORGCHK to check whether index(es) need maintenance.
Ghost records turn out to be useful for a number of purposes.

*E.g., fence keys*

- Keep a copy of parent’s separator keys in every node
  
  Fence keys span range of possible key values in this node
  
  → Avoids problems with prefix truncation.

- One key is an exclusive bound, thus must be a ghost record.

- The other one may or may not be a ghost record.

- Can be used, *e.g.*, to check integrity of B-tree.
Variable-Length Keys

With **variable-length keys**, the original B-tree property

\[ d \leq \text{number of keys in a node} \leq 2d \]

is not practical any more.

→ Real-world systems do not really care about this “50 % rule.”

With truncation, the storage space for a key might even change during re-organizations.

■ 📝 Will this cause any trouble during updates?
B+-trees can (in theory\textsuperscript{6}) be used to index everything with a defined total order, e.g.:

- integers, strings, dates, . . . , and
- concatenations thereof (based on lexicographical order).

E.g., in most SQL dialects:

```
CREATE INDEX ON TABLE CUSTOMERS (LASTNAME, FIRSTNAME);
```

A useful application are, e.g., partitioned B-trees:

- Leading index attributes effectively partition the resulting B+-tree.


\textsuperscript{6}Some implementations won’t allow you to index, e.g., large character fields.
CREATE INDEX ON TABLE STUDENTS (SEMESTER, ZIPCODE);

What types of queries could this index support?
Bulk-Loading $B^+$-trees

Building a $B^+$-tree is particularly easy when the input is sorted.

- Build $B^+$-tree **bottom-up** and **left-to-right**.
- Create a parent for every $2d + 1$ unparented nodes.
  (Actual implementations typically leave some space for future updates. e.g., DB2’s PCTFREE parameter)

What use cases could you think of for bulk-loading?
In the foregoing we described the B$^+$-tree.

Bayer and McCreight originally proposed the B-tree:
- Inner nodes contain data entries, too. 

There is also a B*-tree:
- Keep non-root nodes at least $2/3$ full (instead of $1/2$).
- Need to redistribute on inserts to achieve this. (Whenever two nodes are full, split them into three.)

Most people say “B-tree” and mean any of these variations. Real systems typically implement B$^+$-trees.

“B-trees” are also used outside the database domain, e.g., in modern file systems (ReiserFS, HFS, NTFS, ...).
B$^+$-trees are **by far** the predominant type of indices in databases. An alternative is **hash-based indexing**.

- Hash indices can only be used to answer **equality predicates**.
- Particularly good for strings (even for very long ones).
Dynamic Hashing

**Problem:** How do we choose $n$ (the number of buckets)?

- $n$ too large $\rightarrow$ space wasted, poor space locality
- $n$ too small $\rightarrow$ many overflow pages, degrades to linked list

Database systems, therefore, use **dynamic hashing** techniques:

- extendible hashing,
- linear hashing.

Few systems support true hash indices (e.g., PostgreSQL).

More popular uses of hashing are:

- support for $\mathbf{B^+}$-trees over hash values (e.g., SQL Server)
- the use of hashing during query processing $\rightarrow$ **hash join**.
Recap

Indexed Sequential Access Method (ISAM)
A static, tree-based index structure.

$\mathbf{B^+}$-trees
The database index structure; indexing based on any kind of (linear) order; adapts \textit{dynamically} to inserts and deletes; low tree heights ($\sim 3–4$) guarantee fast lookups.

Clustered vs. Unclustered Indices
An index is clustered if its underlying data pages are ordered according to the index; fast \textit{sequential access} for clustered $\mathbf{B^+}$-trees.

Hash-Based Indices
\textbf{Extendible hashing} and \textbf{linear hashing} adapt dynamically to the number of data entries.