Part IX

B-Trees
Memory Hierarchy

- fast, but expensive and small, memory close to CPU
- larger, slower memory at the periphery
- Try to hide latency by using the fast memory as a cache.
“Slow” memory typically means high latency.

Example: Samsung HD642JJ Hard Drive (640 GB, SATA 3)

- rotational speed: 7200 rpm
- sequential read bandwidth: \( \approx 106 \text{ MB/s} \) (\( \uparrow \text{ hdparm -t} \))
- random access time: 15.2 ms (measured)

🔗 Time it takes to read 1,024 random 4 kB blocks?

```plaintext
1024 \times 15.2 \text{ ms random access time} = 15.56 \text{ s}
4 \text{ MB} \div 106 \text{ MB/s transfer time} = 0.04 \text{ s}
```

Total time: 15.60 s
Ways to Improve I/O Performance

The latency penalty is hard to avoid.

However:
- Throughput can be increased rather easily by exploiting parallelism.
- Idea: Use multiple disks and access them in parallel.

TPC-C: An industry benchmark for OLTP

The current number one system (Oracle 11g RAC on SPARC) uses
- 11,040 flash drives (24 GB each) and 720 hard drives (!) (plus drives for OS, etc.),
- connected with 8 Gbit Fibre Channel,
- yielding 30 tpmC (≈ 60 M transactions per minute).
Consequences of the Bandwidth $\leftrightarrow$ Latency Gap

To combat the latency problem:

1. Databases access and organize the disk with a **page granularity**.
   - Read larger chunks to amortize high latency.
   - Page size: at least 4 kB, better more; up to $\approx 64$ kB.

2. Use **sequential access** and/or aggressive **prefetching** (read-ahead).
   - But must read **many** pages ahead to actually avoid penalty.
To answer this query, we could

1. **sort** the table on disk (in ZIPCODE order).
2. To answer queries, then use **binary search** to find first qualifying tuple, and **scan** as long as ZIPCODE < 8999.

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$k*$ denotes the full data record with search key $k$. 
Need to read only $\log_2(\# \text{ tuples})$ to find the first match.

Need to read about as many pages for this.

(The whole point of binary search is that we make far, unpredictable jumps. This largely defeats prefetching.)
Observations:

- Make rather **far jumps initially**.
  - For each step read **full page**, but inspect only **one record**.
- Last $O(\log_2 \text{pagesize})$ steps stay **within one page**.
  - I/O cost is used much more efficiently here.
**Idea:** “Cache” those records that might be needed for the first phase.

→ If we can keep the cache in memory, we can find any record with just a single I/O.

️ **Is this assumption reasonable?**
What if my data set is really large?

- “Cache” will span many pages, too. (In practice, we’ll organize the cache just like any other database object.)
- Thus: “cache the cache” → hierarchical “cache”

B-trees are essentially such a “hierarchical cache.”
B-Trees

- All nodes are the size of a page
  - hundreds of entries per page
  - large fanout, low depth
- Search effort: $\log_{\text{fanout}}(\# \text{ tuples})$

B-Trees

Each B-tree node contains

- A set of **index entries**, which include
  - the value of a **search key** (e.g., 4711) and
  - “**associated information**” (indicated by *)
    (either a full data tuple or a reference to a tuple).
- A set of **child pointers**, pointing to a child page of the B-tree.

Each tree node (except the root) contains **at least** $d$ and **at most** $2d$ index entries (“minimum 50 % full”; on previous slide: $d = 2$).

- We call $d$ the **order** of the B-tree.
- In practice, $d$ is **large** (few hundreds).

B-trees are **balanced** at all times.
Searching a B-Tree

Function: \( \text{tree\_search}(k, node) \)

1. if matching \( *i \) found on \( node \) then
2. \hspace{1cm} return \( *i \);
3. if \( node \) is a leaf node then
4. \hspace{1cm} return not found;
5. switch \( k \) do
6. \hspace{2cm} case \( k < k_0 \) do
7. \hspace{3cm} return \( \text{tree\_search}(k, p_0) \);
8. \hspace{2cm} case \( k_i < k < k_{i+1} \) do
9. \hspace{3cm} return \( \text{tree\_search}(k, p_i) \);
10. \hspace{2cm} case \( k_{2d} < k \) do
11. \hspace{3cm} return \( \text{tree\_search}(k, p_{2d}) \);

Invoke with

\( node = \) root node.

Note that B-trees are an ordered index structure.

\( \rightarrow \) Support equality and range predicates
B-Tree Modifications

**Goal:** Keep B-tree **balanced** at all times.\(^{14}\)

Why is this desirable?

**Thus:** Define routines for **insertion** and **deletion** that leave the B-tree properties intact.

\(^{14}\) *i.e.*, every root-to-leaf path must have the same length.
Inserting into a B-Tree

Sketch of the insertion procedure for entry $k*$:

1. **Find leaf page** $n$ where we would expect the entry for $k$.

2. If $n$ has **enough space** to hold the new entry (i.e., at most $2d - 1$ entries in $n$), **simply insert** $k*$ into $n$.

3. Otherwise node $n$ must be **split** into $n$ and $n'$ and a new **separator** has to be inserted into the parent of $n$.

Splitting happens recursively and may eventually lead to a split of the root node (increasing the tree height).

→ **B-trees grow at the root, not at the leaves!**
Insert new entry with key 4222.

→ Enough space in node 3, simply insert.
B-Tree Insert: Examples (Insert with Leaf Split)

Insert key 6330.

→ Must split node 4.

→ Middle entry goes into node 1.
B-Tree Insert: Examples (Insert with Inner Node Split)

After 8180, 8245, 6435 insert key 4104.

→ Must split node 3.
→ Node 1 overflows → split it
→ New separator goes into root
Insert: Root Node Split

- Splitting starts at the leaf level and continues upward as long as index nodes are fully occupied.
- Eventually, this can lead to a split of the root node:
  - Split like any other inner node.
  - Use the separator to create a new root.
- The root node is the only node that may have an occupancy of less than 50%.
- This is the only situation where the tree height increases.

How often do you expect a root split to happen?
A B-tree maintains key values together with “associated information”.

The “associated information” can be

**Full Data Tuples**

The B-tree becomes the mechanism to organize the table data

→ The table is physically ordered according to the key attribute.
→ We call this a clustered index or an index-organized table.
→ There can be at most one clustered index per table.

**Pointers to Actual Tuples**

These pointers are also called record identifiers or RIDs.

→ Most systems use \( \langle \text{pageno}, \text{pos. within page} \rangle \) to encode RIDs.
→ Such indexes are also called secondary indexes.
→ There can be arbitrarily many secondary indexes.

Many systems (e.g., DB2) only support the latter index type.
Key to the efficiency of B-trees is their high fanout.

- high fanout $\rightarrow$ low tree depth $\rightarrow$ fast root-to-leaf navigation

This gives incentive to maximize fanout:
- Do not store $*$ in inner nodes
  (Rather use that space to increase $d$ / store more keys.)
- Inner nodes are then used for root-to-leaf navigation only.
- For every data tuple, there is on leaf-level index entry.
- The resulting index structure is then called $B^+$-tree.

Real systems today always use $B^+$-trees.
- When database people say “B-tree,” they typically mean “$B^+$-tree.”
**B⁺-trees**

- Inner nodes do **not** store tuples or RIDs
  - only used to navigate to leaves
  - higher fanout, lower depth
- Only leaves contain (references to) tuple data (indicated here with *)
### Searching a B$^+$-tree

1. **Function:** `search(k)`
   
   ```
   return tree_search(k, root);
   ```

2. **Function:** `tree_search(k, node)`
   
   ```
   if node is a leaf then
       return node;
   switch k do
       case $k \leq k_0$ do
           return tree_search(k, $p_0$);
       case $k_i < k \leq k_{i+1}$ do
           return tree_search(k, $p_i$);
       case $k_{2d} < k$ do
           return tree_search(k, $p_{2d}$);
   ```

- All searches now navigate to a leaf node.
  → Makes search effort also more predictable.
- Function `search(k)` returns a pointer to the leaf node that contains potential hits for search key $k$. 

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B+-tree Insert: Examples (Insert without Split)

Insert new entry with key \textbf{4222}.

→ Enough space in node 3, simply insert.
(Same as in B-tree)
Insert key 6330.

→ Must split node 4.

→ New separator goes into node 1.
But keep entry in node 4!
B\textsuperscript{+}-tree Insert: Examples (Insert with Inner Node Split)

After 5219, 5476, insert key 4104.

→ Must \textbf{split} leaf node 3.

→ Inner node 1 overflows → split it

→ \textbf{New separator} goes into root

Splitting the \textbf{inner node} works analogously to B-tree.
B$^+$-tree Insertion Algorithm

Function: tree_insert \((k, rid, node)\)

1. if node is a leaf then
   1. return leaf_insert \((k, rid, node)\);

else

   switch \(k\) do
     2. case \(k \leq k_0\) do
         3. \(<sep, ptr> \leftarrow \text{tree_insert}(k, rid, p_0)\);

     4. case \(k_i < k \leq k_{i+1}\) do
         5. \(<sep, ptr> \leftarrow \text{tree_insert}(k, rid, p_i)\);

     6. case \(k_{2d} < k\) do
         7. \(<sep, ptr> \leftarrow \text{tree_insert}(k, rid, p_{2d})\);

   if \(sep\) is null then
     8. return \(<null, null>\);

   else
     9. return split \((sep, ptr, node)\);


see tree_search ()
Function: leaf_insert \((k, rid, node)\)

if another entry fits into node then
  insert \(<k, rid>\) into node;
  return \(<null, null>\);
else
  allocate new leaf page \(p\);
  take \(\{<k_1^+, p_1^+>, \ldots, <k_{2d+1}^+, p_{2d+1}^+>\}\) := entries from node \(\cup\) \(\{<k, ptr>\}\)
  leave entries \(<k_1^+, p_1^+>, \ldots, <k_{d+1}^+, p_{d+1}^+>\) in node;
  move entries \(<k_{d+2}^+, p_{d+2}^+>, \ldots, <k_{2d+1}^+, p_{2d+1}^+>\) to \(p\);
  return \(<k_{d+1}^+, p>\);

Function: split \((k, ptr, node)\)

if another entry fits into node then
  insert \(<k, ptr>\) into node;
  return \(<null, null>\);
else
  allocate new leaf page \(p\);
  take \(\{<k_1^+, p_1^+>, \ldots, <k_{2d+1}^+, p_{2d+1}^+>\}\) := entries from node \(\cup\) \(\{<k, ptr>\}\)
  leave entries \(<k_1^+, p_1^+>, \ldots, <k_d^+, p_d^+>\) in node;
  move entries \(<k_{d+2}^+, p_{d+2}^+>, \ldots, <k_{2d+1}^+, p_{2d+1}^+>\) to \(p\);
  set \(p_0 \leftarrow p_{d+1}^+\) in node;
  return \(<k_{d+1}^+, p>\);
B+-tree Insertion Algorithm

1  Function: insert (k, rid)
2  ⟨key, ptr⟩ ← tree_insert (k, rid, root);
3  if key is not null then
4    allocate new root page r;
5    populate n with
6      p0 ← root;
7      k1 ← key;
8      p1 ← ptr;
9    root ← r;

- insert (k, rid) is called from outside.
- Note how leaf node entries point to RIDs, while inner nodes contain pointers to other B+-tree nodes.
**Example:** Webserver access log (people.inf.ethz.ch)
- table cardinality: 11 million tuples (710K data pages)
- distinct IP addresses: 181,628 (stored as CHAR (15))
- database: IBM DB2 9.7

**B⁺-tree on IP addresses:**
- 25,151 index pages total:
  - 1 root node
  - 110 second-level nodes; average fanout: 230
  - 25,040 leaf-level nodes: 1–77 keys per node
Deletion

- If a node is sufficiently full (i.e., contains at least $d + 1$ entries, we may simply remove the entry from the node.
  - Note: Afterward, **inner nodes** may contain keys that no longer exist in the database. This is perfectly legal.

- **Merge** nodes in case of an **underflow** ("undo a split"):

  ![Diagram](image.png)

  - “Pull” separator into merged node.
Deletion

It’s not quite that easy...

- Merging only works if **two** neighboring nodes were 50% full.
- Otherwise, we have to **re-distribute**:  
  - “rotate” entry through parent
- Redistribution is **complex** and **expensive**.
  → Real systems usually do not implement deletion “by the book.”
- Actual systems often avoid the cost of merging and/or redistribution, but relax the minimum occupancy rule.

- *E.g.*, **IBM DB2 UDB**:
  - The `MINPCTUSED` parameter controls when the system should try a leaf node merge ("on-line index reorg").
  - Inner nodes are never merged (→ need to do full table reorg for that).

- To improve **concurrency**, systems sometimes only mark index entries as deleted and physically remove them later (*e.g.*, IBM DB2 UDB "type-2 indexes")
  - Resulting index entries are also called **ghost records**.
A typical situation (for a secondary $B^+$-tree) looks like this:

What are the implications when we want to execute

```
SELECT * FROM CUSTOMERS ORDER BY ZIPCODE
```

"Random" access to data pages when we scan the $B^+$-tree.

Page I/Os needed: $\approx$ number of tuples in `CUSTOMERS`.

For comparison: Using external sorting, we could sort the entire file with 3–5 sequential file reads.
B$^+$-trees can (in theory$^{15}$) be used to index everything with a defined total order, e.g.:

- integers, strings, dates, . . . , and
- concatenations thereof (based on lexicographical order).

E.g., in most SQL dialects:

```sql
CREATE INDEX ON TABLE CUSTOMERS (LASTNAME, FIRSTNAME);
```

A useful application are, e.g., partitioned B-trees:

- Leading index attributes effectively partition the resulting B$^+$-tree.


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$^{15}$Some implementations won’t allow you to index, e.g., large character fields.
CREATE INDEX ON TABLE STUDENTS (SEMESTER, ZIPCODE);

What types of queries could this index support?